THE ELECTROMAGNET

UNDERHILL

D. VAN NOSTRAND COMPANY
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JOSEPH HENRY

THE ELECTROMAGNET

BY

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PREFACE.

THIS book is a new and revised edition of "The Electromagnet" by Townsend Wolcott, A. E. Kennelly, and Richard Varley.

The author has endeavored to give the facts connectedly, so that the reader may easily follow the reasoning without referring to different parts of the book, with the exception of the tables, which are placed in the Appendix for convenience.

Much of the data, especially that concerning windings, has been obtained from actual practice.

As the economy and efficiency of an electromagnet depend largely on the proper design and calculation of the winding, particular attention has been paid to that detail.

Acknowledgments are due to R. Varley, W. J. Varley, A. D. Scott, J. M. Knox, and W. H. Balke for data and assistance.

C. R. UNDERHILL.

PROVIDENCE, R.I., August 29, 1903.

CONTENTS

CHAPTER II.

WINDING CALCULATIONS.

CHAPTER III.

HEATING OF MAGNET COILS.

CHAPTER IV.

ELECTROMAGNETS AND SOLENOIDS.

CHAPTER V.

ELECTROMAGNETIC PHENOMENA.

APPENDIX.

NOTATION.

a = percentage of copper in cotton insulated wire.
a_1 = percentage of copper in silk insulated wire.
A = area in square inches.
A_c = area in square centimeters.
b = distance between centers of cores in inches.
B = magnetic induction (English system).
$\mathcal{B}$ = magnetic induction in gausses.
c = constant = .0000027107.
C.M. = circular mils.
C_w = weight of cotton in pounds.
d = diameter of core + sleeve.
$\left.\begin{matrix} d_1 \\ d_2 \end{matrix}\right\}$ = as in Fig. 36, p. 76.
$\left.\begin{matrix} d_3 \\ d_4 \end{matrix}\right\}$ = as in Fig. 41, p. 80.
d_5 = as in Fig. 42, p. 82.
d_c = diameter of core.
d^θ = deflection of galvanometer.
D = true outside diameter of round windings.
$\left.\begin{matrix} D_1 \\ D_2 \end{matrix}\right\}$ = as in Fig. 36, p. 76.
$\left.\begin{matrix} D_3 \\ D_4 \end{matrix}\right\}$ = as in Fig. 41, p. 80.
D_5 = as in Fig. 42, p. 82.
E = E.M.F. = electromotive force in volts.
e = base of Naperian logarithms = 2.7182818.

F = magnetomotive force (English system).
$\mathfrak{F}$ = M.M.F. = magnetomotive force in gilberts.
f = number of cycles per second.
g = total diameter of insulated wire.
g^2 = space factor.
g_l = lateral value of wire and insulation.
g_v = vertical value of wire and insulation.
H = magnetizing force (English system).
$\mathfrak{H}$ = magnetizing force in gausses.
H = as in Fig. 42, p. 82.
H.P. = horse-power.
I = current in amperes.
IN = ampere-turns.
Jr = joint resistance.
K = resistance factor = $\pi\,\omega''$.
k = constant of galvanometer.
L = length of winding.
Lb. = pounds adv.
l = length of magnetic circuit in inches.
l_c = length of magnetic circuit in centimeters.
l_p = length of wrap of paper in inches.
l_w = length of wire or strand in inches.
$\mathbf{L}$ = inductance in henries.
M = mean or average diameter of winding in inches.
M_1 = as in Fig. 42, p. 82.
m = turns of wire per inch.
N = number of turns of wire in winding.
n = number of layers.
n_c = a constant (see p. 35).
n_w = number of wires.
P = paper allowance for duplex windings.

P = magnetic attraction or pull.
R = combined resistance and space factor $= \frac{K}{g^2}$.
$\mathcal{R}$ = magnetic reluctance in oersteds.
$\mathbf{R}$ = magnetic reluctance (English system).
r = radius of circle.
s = gauge number of wire (B. & S.).
S = silk allowance for duplex windings.
S_r = radiating surface in square inches.
S_w = weight of silk in pounds.
T = thickness or depth of winding.
t = time constant.
t° = rise in temperature.
V = volume of winding in cubic inches.
V_l = leakage coefficient.
V_p = volume of paper in duplex windings (cubic inches).
V_s = volume of silk space in cubic inches.
w = combined weight and space factor $= \frac{R}{\theta}$.
W = watts.
W_s = watts per square inch.
W_c = watts lost per cubic centimeter of iron.
x = as in Fig. 15, p. 16.
X = intermediate diameter in inches.
Z = impedance.
Δ = diameter of wire in inches.
θ = ohms per pound for insulated wires.
λ = weight of bare wire in pounds.
μ = permeability.
π = 3.1416 = ratio between diameter and circumference of circle.
ρ = electrical resistance.
ρ_s = series resistance.

Σ = cross-section of insulation in circular inches.
ϕ = flux in webers = lines of force.
ϕ_1 = useful flux.
ϕ_2 = leakage in webers.
Ω = ohms per pound for bare wires.
ω'' = ohms per inch.
ω' = ohms per foot.

THE ELECTROMAGNET.

CHAPTER I.

ELECTRIC AND MAGNETIC CIRCUIT CALCULATIONS.

1. Magnetism.

"*Magnetism* is that peculiar property occasionally possessed by certain bodies (more especially iron and steel) whereby under certain circumstances they naturally attract or repel one another according to determinate laws."

Magnetism is supposed to have first been discovered by the ancients in Magnesia, Thessaly, where they found an ore which possessed a remarkable tractive power for iron. A piece of the ore having this power they termed a *Magnet.*

It was also discovered that when a piece of this ore was suspended so that it could move freely, one of its ends always pointed to the north, and the other end, of course, pointed south. Navigators took advantage of this principle to steer their ships, and hence the name *Lodestone* (meaning Leading Stone) was given to the natural ore.

Artificial magnets were made by rubbing a bar of hardened steel with a piece of lodestone.

Artificial magnets which retain their magnetism for a long time are called *Permanent Magnets.*

2. Magnetic Poles.

The end of a magnet which has a tendency to point north is naturally termed its *North Pole*, although this arrangement really makes the pole situated near the geographical North Pole of the earth the magnetic South Pole of the earth, since like magnetic poles repel one another and unlike poles attract each other.

In this book the term *North-seeking Pole* will be used instead of *North Pole*, as the latter is liable to become confused with the geographical North Pole of the earth.

The north-seeking pole of a magnet is equal in strength to its south-seeking pole, the strength gradually decreasing until midway between the poles there is no attraction at all. This place is called the *Neutral Line*.

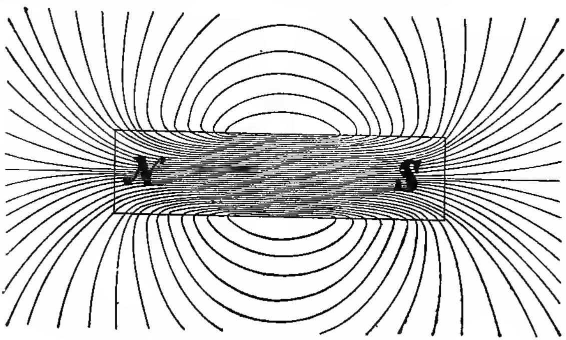


Fig. 1.

Every magnet has two poles, and if the *Bar Magnet* in Fig. 1 should be broken into any number of pieces, each piece would be a perfect magnet, with a north-seeking and south-seeking pole of equal strength.

3. Magnetic Field.

If a piece of paper be laid over a bar magnet and iron filings sprinkled over the paper, and then if the paper be jarred slightly so as to give the filings an opportunity to settle freely, they will take positions as shown in Fig. 2.

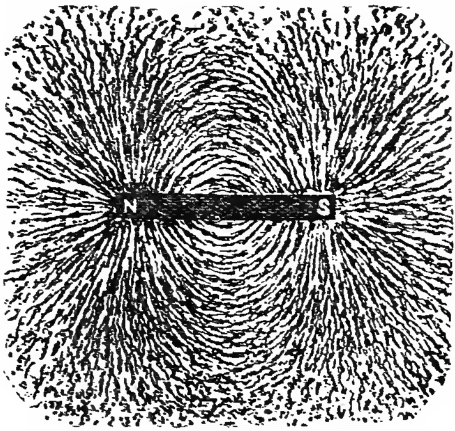


Fig. 2.

From mathematical and experimental research it has been found that the magnetism passes through the inside of a magnet from pole to pole, issuing from its north-seeking pole, and returning through the air or surrounding media to its south-seeking pole, although all of the magnetism does not pass from the ends of the magnet (as may be seen by reference to Fig. 2), which fact shows that all of the magnet on one side of the center, or neutral line, is north-seeking, while all on the other side is south-seeking, the poles being stronger near the ends.

These streams of magnetism are called *Lines of Force*, and the media about the poles of the magnet, through which they pass, is called the *Field of Force*. The lines are always closed curves; hence the path through which they flow is called the *Magnetic Circuit*.

4. Forms of Permanent Magnets.

The usual form of the magnetic circuit is substantially as shown in Fig. 3. This form is called a *Horseshoe* permanent magnet, and is a bar bent into the shape of a horseshoe. This is done to bring the two poles of the magnet near each other, and thus shorten the magnetic circuit. The piece of iron which is attracted by the magnet is called its *Armature*.

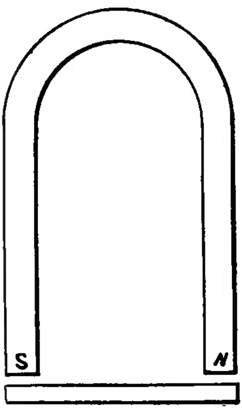


Fig. 3.

The lines of force flow out through the north-seeking pole of the magnet, through the armature, and into the south-seeking pole, through the substance of the magnet to the starting-point.

To obtain the largest number of lines of force through a magnet, the magnetic circuit should be as short, and have as few *Air Gaps*, as possible.

Another form is shown in Fig. 4, and in effect is merely two horseshoe magnets placed with similar poles together, thus tending to repel one another. A magnet thus arranged is said to have *Consequent Poles*. The same results would be obtained if two or more horseshoe

magnets were placed side by side with similar poles together.

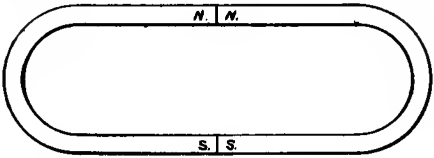


Fig. 4.

Magnets of the latter form are called *Compound Magnets*, and are commonly used on magneto generators. (See Fig. 5.)

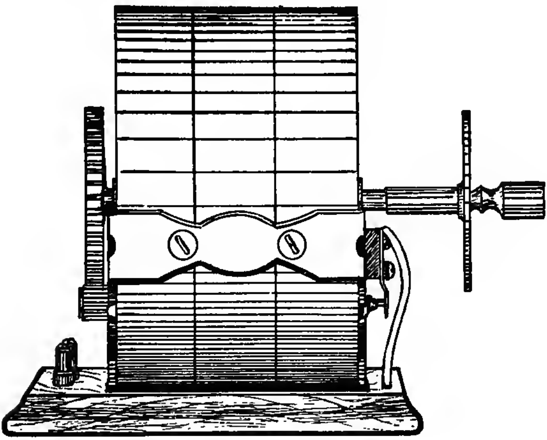


Fig. 5.

In this type of magnet it is very important that each of the separate magnets should have the same strength; otherwise, the weaker magnets would act as a return cir-

cuit for the stronger ones, and the effective field would thus be weakened.

5. Magnetic Induction.

When a magnet attracts a piece of iron, this iron itself becomes a magnet while it is being attracted, and will attract other pieces of iron, which in turn also become magnets. This successive magnetization of the iron pieces is said to be produced by *Magnetic Induction.*

6. Electric Circuit.

The force which causes a current of electricity to flow through a conductor is called *Electromotive Force* (abbreviated E.M.F.), and the unit used in practice is the *Volt.*

Every known substance offers some *Resistance* to the passage of an electric current. The practical unit of electrical resistance is the *Ohm.*

The unit strength of electric current is the *Ampere* and is produced by the unit electromotive force acting through the unit resistance.

The rule expressing the relation between electromotive force, current strength, and resistance is known as

7. Ohm's Law.

The strength of the current is equal to the electromotive force divided by the resistance, or

$$I = \frac{E}{\rho}. \tag{1}$$

Transposing,

$$E = I\rho \tag{2}$$

and $$\rho = \frac{E}{I}, \tag{3}$$

where $I =$ strength of current,
$E =$ electromotive force,
$\rho =$ resistance.

The unit of electric power is termed the *Watt.* *The watts are equal to the square of the current multiplied by the resistance*, or $$W = I^2\rho, \tag{4}$$

whence, by substitution,

$$W = \frac{E^2}{\rho} \text{ (5), also, } \quad W = EI. \tag{6}$$

746 watts equal one horse-power, or

$$\text{H.P.} = 746\ W. \tag{7}$$

Therefore, one watt equals .00134 horse-power, or

$$W = .00134 \text{ H.P.} \tag{8}$$

8. Divided or Branched Circuits.

When any number of equal resistances are connected in multiple, the *Joint Resistance* is equal to the resistance of one conductor divided by the number of conductors.

The joint resistance of two equal or unequal resistances connected in multiple is equal to their product divided by their sum, or

$$Jr = \frac{\rho\rho_1}{\rho + \rho_1}. \tag{9}$$

EXAMPLE. — The resistance of two electromagnets is 74 ohms and 92 ohms respectively. What will be the joint resistance when they are connected in multiple?

SOLUTION. —

$$\frac{74 \times 92}{74 + 92} = \frac{6,808}{166} = 41 + \text{ ohms.} \quad \textit{Ans.}$$

The joint resistance of three or more conductors in multiple is equal to the reciprocal of their joint conductivity.

Since the conductivity of a conductor is the reciprocal of its resistance, the conductivity $= \frac{1}{\rho}$. Therefore, if we let ρ_1, ρ_2, and ρ_3 equal the separate resistances of the three branches, as in Fig. 6, the conductivities will be $\frac{1}{\rho_1}$, $\frac{1}{\rho_2}$, and $\frac{1}{\rho_3}$ respectively.

Their joint conductivity is

$$\frac{1}{\rho_1} + \frac{1}{\rho_2} + \frac{1}{\rho_3} = \frac{\rho_2\rho_3 + \rho_1\rho_3 + \rho_1\rho_2}{\rho_1\rho_2\rho_3},$$

and the reciprocal

$$= \frac{\rho_1\rho_2\rho_3}{\rho_2\rho_3 + \rho_1\rho_3 + \rho_1\rho_2} = Jr, \tag{10}$$

which is the same as (9).

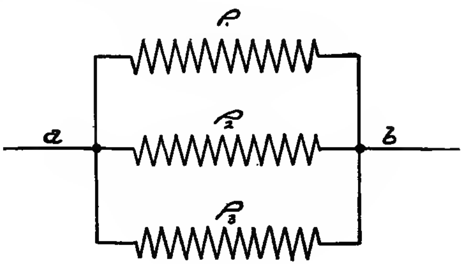


Fig. 6.

EXAMPLE. — Three electromagnets having resistances of 4 ohms, 6 ohms, and 8 ohms respectively are to be connected in multiple. What will be their joint resistance?

SOLUTION. — By formula (10),

$$Jr = \frac{\rho_1\rho_2\rho_3}{\rho_2\rho_3 + \rho_1\rho_3 + \rho_1\rho_2} = \frac{4 \times 6 \times 8}{(6 \times 8) + (4 \times 8) + (4 \times 6)}$$
$$= \frac{192}{104} = 1.84+ \text{ ohms.}$$

The current which will flow through each branch of the circuit is found by ascertaining the total current flowing through the branches, and then by formula (2), $E = I\rho$, find the electromotive force across the branches from a to b, and next by applying formula (1), $I = \frac{E}{\rho}$, find the current flowing through each branch.

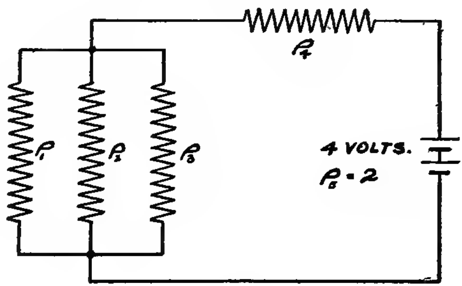


Fig. 7.

EXAMPLE. — In the diagram (Fig. 7), $\rho_1 = 3$ ohms, $\rho_2 =$ 4 ohms, $\rho_3 = 5$ ohms, $\rho_4 = 1$ ohm, and ρ_5, the internal resistance of the battery $= 2$ ohms.

How many amperes of current will flow through each branch?

SOLUTION. — By formula (10) the joint resistance of the branched circuit is 1.28 ohms nearly. By Ohm's law, formula (1), the current

$$I = \frac{E}{\rho} = \frac{4}{1.28 + 1 + 2} = \frac{4}{4.28} = .935 \text{ amperes.}$$

By formula (2) the electromotive force across the branched circuit from a to $b = .935 \times 1.28 = 1.2$ volts nearly. Then by formula (1)

$$I_1 = \frac{1.2}{3} = .4, \qquad I_2 = \frac{1.2}{4} = .3, \qquad I_3 = \frac{1.2}{5} = .24. \quad \textit{Ans.}$$

Fig. 8.

From the foregoing it is seen that two resistances, to be connected in multiple and produce the same resistance to the line as if they were connected in series, must have the resistance of each increased four times.

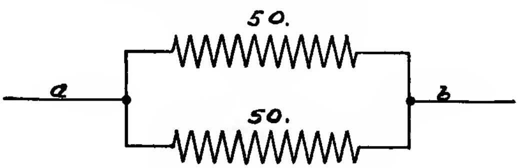


Fig. 9.

Assume two electromagnet windings of 50 ohms each to be used in series, having a total resistance of 100 ohms

as in Fig. 8. If they were connected in multiple as in Fig. 9, the resistance would be $\frac{50}{2} = 25$ ohms, or only $\frac{1}{4}$ of what the resistance would be were they connected in series.

9. Magnetic Units.

In the following discussion it is assumed that there is no magnetic material in the circuit, except when the contrary is stated.

Unit Magnetic Pole in the C.G.S. system is so defined that when placed at one centimeter distance from an exactly similar pole it repels it with a force of one *dyne.*

Now, if a unit magnetic pole be placed at the center of a sphere of one centimeter radius (two centimeters in diameter), there will radiate from this pole one line of force for each square centimeter of surface on the sphere, and as the area of the sphere is equal to $4\pi r^2$ square centimeters, there will be $4 \times 3.1416 \times 1^2 = 12.5664$ lines of force radiating from the unit pole.

One line of force (also called one *Weber*, symbol ϕ) per square centimeter is called unit *Intensity* or unit *Density* of magnetization, and is termed the *Gauss* (symbol $\mathcal{H}$). Thus, 100 gausses are 100 webers per square centimeter. It is to be observed that the gauss has nothing to do with the total area, but it is the number of lines of force *Per Unit Area* in square centimeters.

The number of gausses are found by dividing the total number of lines of force by the total cross-sectional area of the magnet in square centimeters.

The force producing the flow of magnetism is called *Magnetomotive Force*, and the unit is the *Gilbert* (symbol $\mathcal{F}$).

The number of gilberts per centimeter length of magnetic circuit is called the *Magnetizing Force.*

The law of the magnetic circuit is identical with that of the electric circuit, inasmuch as the *Flow* is equal to the potential difference divided by the resistance. (See Ohm's law, page 6.)

In the case of the magnetic circuit, however, when composed of iron or steel, the magnetic resistance called *Reluctance* changes with the flow of magnetism called *Flux*, or more correctly, with the magnetic density or lines per square centimeter.

The property of the iron or steel which causes this variation is called its *Permeability.*

The *Reluctance* or magnetic resistance *is equal to the length of the magnetic circuit, divided by the product of its cross-sectional area and permeability.*

Thus,
$$\mathcal{R} = \frac{l_c}{A_c \mu} \tag{11}$$

where l_c = length of magnetic circuit in centimeters,
$\mathcal{R}$ = reluctance in *oersteds*,
A_c = cross-sectional area in square centimeters,
μ = permeability.

The magnetomotive force (abbreviated M.M.F.) *in gilberts is equal to the number of lines of force, multiplied by the reluctance.*

Thus,
$$\mathfrak{F} = \phi \mathcal{R}, \tag{12}$$

where $\mathfrak{F}$ = gilberts,
ϕ = webers,
$\mathcal{R}$ = oersteds.

Also,
$$\phi = \frac{\mathfrak{F}}{\mathcal{R}}, \tag{13}$$
$$\mathcal{R} = \frac{\mathfrak{F}}{\phi}. \tag{14}$$

Substituting the value of $\mathcal{R}$ from (11) in (12),

$$\mathfrak{F} = \phi \frac{l_c}{A_c\mu}. \quad (15) \qquad \phi = \frac{\mathfrak{F}}{\left(\frac{l_c}{A_c\mu}\right)}. \quad (16)$$

When the magnetic circuit consists of several parts, the total reluctance is equal to the sum of all of the reluctances; thus,

$$\mathcal{R} = \frac{l_{c1}}{A_{c1}\mu_1} + \frac{l_{c2}}{A_{c2}\mu_2} + \frac{l_{c3}}{A_{c3}\mu_3}, \text{ etc.} \tag{17}$$

10. Electromagnetism.

In 1819, Öersted discovered that if a compass needle be brought near a wire carrying an electric current, it tends to take up a position at right angles to the direction of the wire.

The relation which exists between direction of current and deflection of compass needle is as follows: *If the current flows through the wire from left to right, and the needle is above the wire, the north-seeking pole is deflected toward the observer. If below the wire, the north-seeking pole is deflected from the observer.*

11. Force about a Wire.

The explanation of the foregoing is that the wire carrying the current is surrounded by concentric circles of

force, and the compass needle, being a magnet, tends to set itself in the direction of these lines of force. This is illustrated in Fig. 10.

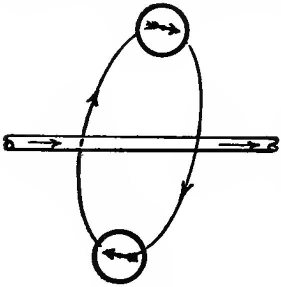


Fig. 10.

The compass needle can never set itself exactly in the direction of the lines of force on account of the earth's magnetism, unless the earth's magnetism be neutralized.

Fig. 11 also shows the relation between direction of current and direction of lines of force. The relation between the current in the wire and the intensity of magnetization, or density of lines of force, is illustrated in Fig. 12.

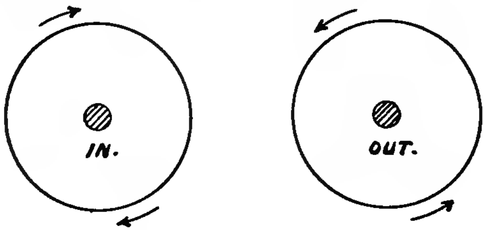
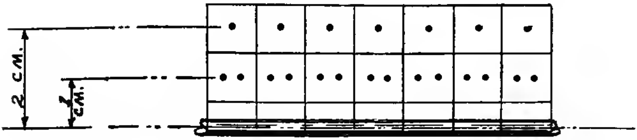


Fig. 11.

When the wire carries 10 amperes, at one centimeter from the center of the wire there are two lines of force (webers) per square centimeter for each centimeter length of wire — that is, two gausses; and at two centimeters from the center of the wire there is but one line of force per square centimeter — that is, there is but one gauss.

Hence the following law: *The intensity in gausses in air is equal to two-tenths times the current in amperes flow-*



10 AMPERES FLOWING THROUGH WIRE.

Fig. 12.

ing through the wire, divided by the distance from the center of the wire in centimeters, or

$$\mathcal{H} = \frac{.2\,I}{a}. \tag{18}$$

The magnetomotive force is equal to the gausses per centimeter length. Thus,

$$\mathcal{F} = \mathcal{H}l_c. \tag{19}$$

Consider the M.M.F. due to the magnetizing force in Fig. 12, but looking at the end of the wire as in Fig. 13.

Here the wire is carrying 10 amperes. Hence at one centimeter from the center of the wire there are two gausses, and at two centimeters from the center of the wire there is but one gauss.

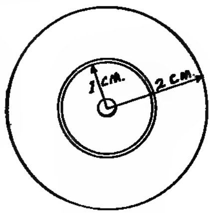


Fig. 13.

Now, at one centimeter, the circumference of the ring of force is 6.2832 centimeters, and the M.M.F. is equal to the number of gausses per centimeter length $= 2 \times 6.2832 = 12.5664$ gilberts.

At two centimeters, the circumference is 12.5664 centimeters, and the M.M.F. is $1 \times 12.5664 = 12.5664$ gilberts. Therefore, the total M.M.F. is always 12.5664 gilberts when 10 amperes is flowing through the wire.

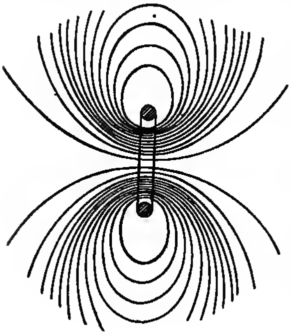


Fig. 14.

When the wire is bent into a circle and a current passed through it, the lines of force are no longer simple circles but are distorted, assuming positions as shown in Fig. 14, so that the force at any point can only be calculated by means of the higher mathematics, but in the center, the intensity

$$\mathcal{H} = \frac{.2\,\pi I}{r}, \tag{20}$$

where r is the radius of the loop or turn of wire as in Fig. 15.

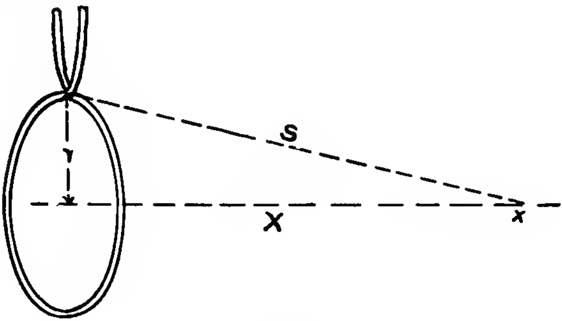


Fig. 15.

At any distance x from the center of loop, on the axis X, the force is

$$\mathcal{H} = \frac{.2\,\pi I r^2}{(r^2 + x^2)^{\frac{3}{2}}}, \tag{21}$$

but at points off the axis, it cannot be calculated by simple algebra, but is approximately uniform near the center of the loop, increasing somewhat toward the wire, while very near the wire it is much greater, especially if the diameter of the wire is small as compared with the diameter of the loop.

The magnetomotive force, however, is still $\mathcal{F} = .4\,\pi I$ (22) gilberts, as in the case of the simple circle about a straight wire shown in Fig. 13.

From the above is deduced the following law: *If a wire one centimeter in length be bent into an arc of one centimeter radius, and a current of 10 amperes passed through the wire, at the center of the arc there will be one line of force per square centimeter, i.e., the intensity will be one gauss.*

12. Ampere-Turns.

In practice the wire is wound in spirals on a bobbin, and one turn of wire with one ampere of current flowing through it is called one *Ampere-turn*, and the same relation holds for any number of turns and any number of amperes. One ampere flowing through one hundred turns gives exactly the same results as one hundred amperes flowing through one turn.

The ampere-turns, then, are found by multiplying the number of turns by the current in amperes flowing through the turns.

The symbol for ampere-turns is IN. Where

$I =$ current in amperes,
$N =$ number of turns.

It has already been stated that a unit magnet pole sends out 12.5664 lines of force, and that a force of one dyne is exerted along each one of these lines, or 12.5664 dynes for the 12.5664 lines of force. Now, it may be shown that the force produced by ten ampere-turns is also 12.5664 dynes or 12.5664 gilberts. Therefore, one ampere-turn produces 1.25664 gilberts.

13. Effect of Iron in Magnetic Circuit.

When iron or steel is introduced into the magnetic circuit, the conductivity of the magnetic circuit called *Permeability* is greatly increased. The permeability of air is taken as unity, and since nearly all substances excepting iron and steel have the same permeability as air, only the two latter will be considered.

There is no insulator of magnetism. Lines of force pass through or permeate every known substance.

In order to distinguish the lines per square centimeter in air from lines per square centimeter in iron or steel, the symbol $\mathcal{B}$ is given to the latter, and they are called *Lines of Induction.*

Thus,

$$\mathcal{B} = \mu\mathcal{H}, \tag{23}$$

$$\mu = \frac{\mathcal{B}}{\mathcal{H}}, \tag{24}$$

$$\mathcal{H} = \frac{\mathcal{B}}{\mu}, \tag{25}$$

where $\mathcal{H}$ = gausses in air,
$\mathcal{B}$ = gausses in iron or steel,
μ = permeability.

14. Terms Expressed in English Measure.

Since in America the units used are in English measure, a great many engineers prefer to change the magnetic units into terms of English measure also.

In Metric measure $\mathfrak{F} = 1.25664\ IN.$ (26)

Therefore, in English measure,

$$\mathbf{F} = 3.192\ IN. \tag{27}$$

Unless otherwise specified in what follows, all symbols will represent units in English measure; and in order to avoid confusion the same symbols as applied to the units in metric measure will be used, but in **heavy type** and English characters.

Thus, $\mathbf{F} = 3.192\ IN,$ (27)

$$IN = .3132\ \mathbf{F}, \tag{28}$$

also $\mathbf{B} = \mu \mathbf{H}.$ (23)

15. General Relations between Magnetic Units.

From (15)

$$\mathbf{F} = \phi \frac{l}{A\mu}.$$

Substituting the value of $\mathbf{F}$ in (28)

$$IN = .3132\ \phi \frac{l}{A\mu}. \tag{29}$$

That is, the ampere-turns required to produce the flux ϕ are equal to the product of .3132 times the flux and the reluctance,

also $$\phi = \frac{3.193\, INA\mu}{l}. \tag{30}$$

That is, the total flux is equal to 3.193 times the ampere-turns divided by the reluctance.

The number of lines per square inch is equal to the total flux divided by the cross-sectional area of the magnetic field, or

$$B = \frac{\phi}{A}. \tag{31}$$

Substituting the value of B from (31) in (29),

$$IN = \frac{.3132\, Bl}{\mu} \tag{32}$$

$$= \frac{Bl}{3.193\, \mu}, \tag{33}$$

whence, $$B = \frac{3.193\, IN\mu}{l}. \tag{34}$$

That is, the number of lines per square inch is equal to the product of 3.193 times the ampere-turns and permeability, divided by the total length of the magnetic circuit.

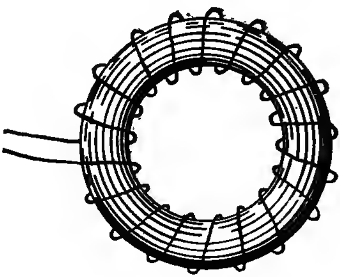


Fig. 16.

The above formulæ apply to the magnetic circuit consisting of a continuous iron or steel ring as in Fig. 16.

16. Permeability.

The permeability μ decreases as the magnetic density B increases, and is found for various grades of iron or steel by actual test, and then curves plotted on charts, or tables made.

(See permeability table on p. 149.)

EXAMPLE. — Assume the mean diameter of the iron ring in Fig. 16 to be three inches, and the cross-sectional area to be .6 square inch, and that it is required to force 60,000 lines of force through the iron. How many ampere-turns are required?

SOLUTION. — Since the mean diameter is three inches, the length of the magnetic circuit is $3 \times 3.1416 = 9.4248$ inches $= l$.

60,000 lines through .6 square inch is equivalent to 100,000 lines per square inch $= B$. Assuming the ring to be made of annealed wrought iron, and referring to formula (32) and substituting the values of B, l, and μ,

$$IN = \frac{.3132 \times 100{,}000 \times 9.4248}{360} = \frac{289{,}000}{360}$$

$$= 805 \text{ ampere-turns.} \quad \textit{Ans.}$$

Since there is a wide variation in the permeability in the same grade of iron or steel, the above result would only be approximate unless a very careful test was made of the ring itself.

17. Magnetic Testing.

One method of doing this is illustrated in Fig. 17.

A is the iron ring to be tested, and is wound with a magnetizing coil *C*, which is in series with a source of current *B*, adjustable rheostat *R*, and double-throw reversing-switch *S*.

A secondary winding *W*, called the exploring coil, is connected to a ballistic galvanometer through an adjustable rheostat *R*.

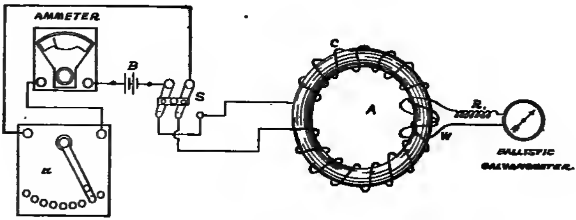


Fig. 17.

The magnetizing force

$$H = \frac{3.192\, IN}{l}, \tag{35}$$

where l is the mean length of the magnetic circuit in the ring.

When the primary circuit is closed or broken, a deflection is produced in the ballistic galvanometer proportional to the magnetic flux.

Then $\quad B = \kappa d^{\theta}, \qquad (36)$

where $\quad d^{\theta}$ = deflection of galvanometer,

κ = constant of galvanometer.

The permeability is then found by formula (24),

$$\mu = \frac{B}{H}.$$

18. Practical Calculations of Magnetic Circuit.

It has been stated that the induction is equal to the magnetizing force multiplied by the permeability, or $B = \mu H$, and the magnetizing force is equal to the M.M.F. per inch. Now, since the M.M.F. is proportional to the ampere-turns, the induction B for any specific case depends upon the ampere-turns per inch of magnetic circuit, and nothing else.

On this principle curves have been constructed which show the value of B for any number of ampere-turns per inch. In Figs. 18 and 19 there are several curves combined representing different grades of iron and steel.

To use the curves, find the point on the curve horizontally opposite the induction per square inch, and then from this point on the curve trace vertically downwards to the ampere-turns per linear inch.

The product of the length of the magnetic circuit into the ampere-turns per linear inch gives the total number of ampere-turns required to maintain the induction B in the iron.

As an example, assume a closed magnetic circuit in the form of a Swedish iron ring, and the average length of the magnetic circuit is 10 inches, and that 87,000 lines per square inch are required.

Referring to the chart, Fig. 18, for 87,000 lines per square inch there are required 20 ampere-turns for each

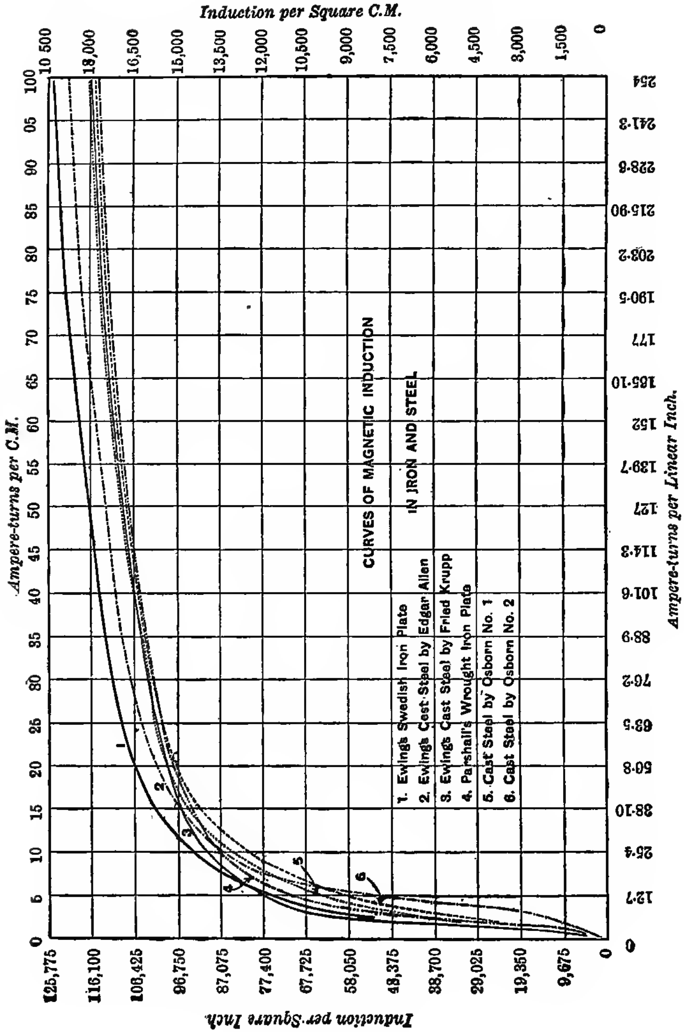

CURVES OF MAGNETIC INDUCTION
IN IRON AND STEEL
1. Ewing's Swedish Iron Plate
2. Ewing's Cast Steel by Edgar Allen
3. Ewing's Cast Steel by Fried Krupp
4. Parshall's Wrought Iron Plate
5. Cast Steel by Osborn No. 1
6. Cast Steel by Osborn No. 2
Induction per Square C.M.
Induction per Square Inch.
Ampere-turns per C.M.
Ampere-turns per Linear Inch.

Fig. 18.

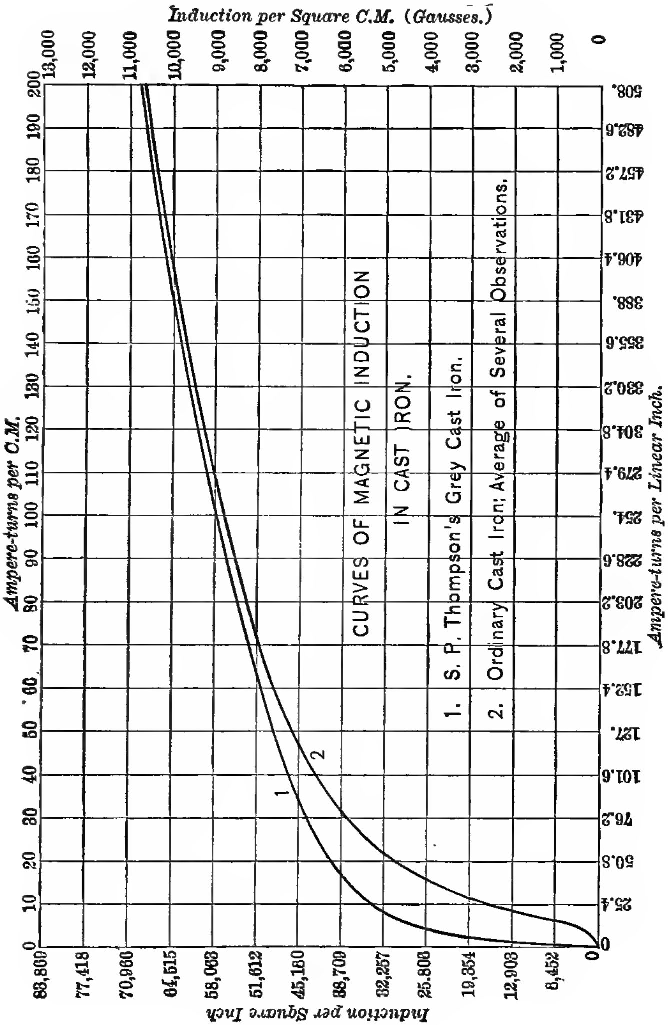

Induction per Square C.M. (Gausses.)
Ampere-turns per C.M.
Ampere-turns per Linear Inch.
Induction per Square Inch
CURVES OF MAGNETIC INDUCTION
IN CAST IRON.
1. S. P. Thompson's Grey Cast Iron.
2. Ordinary Cast Iron; Average of Several Observations.
1
2

Fig. 19.

inch of circuit, and since the length of the circuit is 10 inches, the total ampere-turns required to maintain an induction of 87,000 lines per square inch is $10 \times 20 = 200$.

If the cross-sectional area of the iron ring was .5 square inch, the total number of lines of force would be $87,000 \times .5 = 43,500$.

If the area was 2 inches, the total flux would be $2 \times 87,000 = 174,000$ webers.

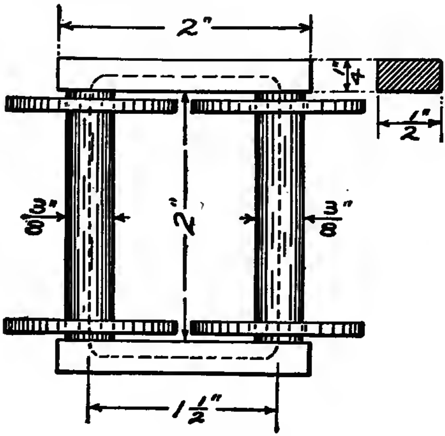


Fig. 20.

When the magnetic circuit consists of the same quality of iron, but of parts of different cross-section, calculate the induction per square inch for one of the parts of the circuit, and then the induction per square inch for the other parts will depend simply on the ratio of their cross-sections to the cross-section of the first part.

Example. — An induction of 100,000 lines per square inch is required in the cores of the magnetic circuit shown in Fig. 20, the cores consisting of Swedish iron. How many ampere-turns are required?

Solution. — The cross-sectional area of the cores is $\frac{3}{8}^2 \times .7854 = .1105$ square inches each. The cross-sectional area of the yoke and armature is $\frac{1}{2} \times \frac{1}{4} = .125$ square inches each.

Since the cross-section of the cores is the smaller, the induction required in the yoke and armature is

$$\frac{100{,}000 \times .1105}{.125} = 88{,}400 \text{ lines per square inch.}$$

Referring to the chart, Fig. 18, the ampere-turns per linear inch required for 100,000 lines per square inch are 34.1, and for 88,400 lines the ampere-turns per inch are 20. The total length of the circuit through the cores is 4 inches, and through the yoke and armature 3.25 inches. Therefore, the ampere-turns required for the cores are $34.1 \times 4 = 136.4$, and for the yoke and armature $20 \times 3.25 = 65$, and the total ampere-turns for the whole magnet $136.4 + 65 = 201.4$. *Ans.*

The lines of induction are nearly straight in the cores, and for that reason the exact length of the cores was used in the calculation. In the yoke and armature, however, the lines bend around something after the form of the dotted lines in Fig. 20, so $1\frac{5}{8}''$ was considered as a fair average for the length of the circuit in the yoke and armature.

In the above example no allowance was made for leakage, or for reluctance at the joints.

If the armature was removed even .001″ from both cores, the total length of the air gap would be .002″, and since the permeability of air is 1, the ampere-turns required for the air gap alone would be

$$IN = \frac{Bl}{3.193} = \frac{100{,}000 \times .002}{3.193} = 62.6,$$

or a total of 264 ampere-turns, including the air gap, an increase of 13.1 per cent over the ampere-turns required for the iron alone.

19. Effect of Joint in Magnetic Circuit.

In addition to increasing the reluctance of the circuit, an air gap introduces leakage and a demagnetizing action due to the influence of the poles induced at the ends of the cores.

The distance between the two faces of an air gap is not the exact length of the gap, as the lines bulge somewhat, as in Fig. 21.

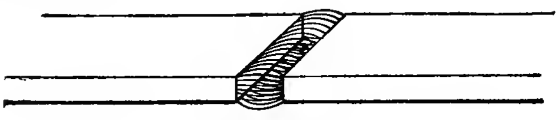


Fig. 21.

Joints or cracks in the magnetic circuit have the same general effect as an air gap, introducing leakage and demagnetization. Ewing and Low found the equivalent air gap for two wrought-iron bars to be about .0012 of an inch.

The effect of the joint is more noticeable for low magnetizations than for high ones, as the increased attraction decreases the distance between the faces of the joint, thus reducing the reluctance.

From the above is seen the importance of having all joints faced as nearly perfect as possible; furthermore, the area of the joint should at least equal the cross-sectional area of the part having the lowest permeability.

20. Magnetic Leakage.

It was stated in art. 13, p. 18, that there is no insulator of magnetism and that the permeability of air is taken as unity.

It is therefore evident that since there is some reluctance in the iron and air gap of the magnetic circuit, some of the lines of force must pass between the cores or other parts of the magnetic circuit, where there is a difference of potential.

The law of the divided electric circuits may be applied to magnetic circuits in this case also, by finding the relative reluctances of the magnetic circuit proper and of the path of the leakage lines.

The leakage may be calculated with great accuracy by plotting the probable leakage paths.

The ratio between the total number of lines generated and the number of useful lines is called the *leakage coefficient*, and is denoted by the symbol V_l. Thus,

$$V_l = \frac{\phi}{\phi_1}, \tag{37}$$

where ϕ = total flux,
ϕ_1 = useful flux.

The reluctance of the air space between two flat surfaces is $\mathcal{R} = \frac{l_c}{A_c u}$ (11), but between two cylinders it is *

$$\mathcal{R} = \frac{.737 \log_{10} \frac{a}{a_1}}{l_c}, \quad (38), \text{ where } \frac{a}{a_1} = \frac{d_c}{b - \sqrt{b^2 - d_c^2}},$$

where d_c = diameter of the cylinders,
b = distance between centers,
l_c = length of cylinders.

The numerical value of $\frac{a}{a_1}$ is constant for all dimensions as long as the ratio $\frac{b}{d_c}$ is constant.

The following table † gives the magnetic reluctance per inch between unit lengths of two equal parallel cylinders surrounded by air and having various values of the ratio $\frac{b}{d_c}$.

$\frac{b}{d_c}$.	R Per Inch.	$\frac{b}{d_c}$.	R Per Inch.	$\frac{b}{d_c}$.	R Per Inch.
1.25	.075	4.0	.258	7.5	.338
1.50	.118	4.5	.264	8.0	.346
1.75	.133	5.0	.287	8.5	.354
2.00	.165	5.5	.299	9.0	.362
2.50	.197	6.0	.311	9.5	.370
3.00	.219	6.5	.321	10.0	.378
3.50	.240	7.0	.331		

* Jackson's Electro-Magnetism and the Construction of Dynamos, p. 131.

† Calculated from table in Jackson's Electro-Magnetism and the Construction of Dynamos.

To find the air reluctance between two equal parallel cylinders, find the ratio $\frac{b}{d_c}$ and opposite this value in the table is the reluctance per inch of length. Therefore, the total reluctance between them is the reluctance per inch divided by the length of each cylinder in inches.

Since at the yoke the difference of magnetic potential is approximately zero, and at the poles it is approximately maximum, the average difference of magnetic potential or M.M.F. is equal to the total M.M.F. divided by 2, or

$$\text{Average M.M.F.} = \frac{3.193\, IN}{2} = 1.5965\, IN.$$

Therefore, the leakage in webers is

$$\phi_2 = \frac{1.5965\, IN}{R}, \tag{39}$$

R being found in the table as explained above.

From this the leakage coefficient V_l is found.

The leakage may be included in the total reluctance by multiplying the sum of the reluctances by the leakage coefficient.

$$\text{Thus,} \qquad R = V_l\left(\frac{l}{Au} + \frac{l_1}{A_1 u_1} + \frac{l_2}{A_2 u_2}\right), \text{ etc.} \tag{40}$$

EXAMPLE. — What is the leakage coefficient of an electromagnet, the reluctance of the magnetic circuit, including the air gap, being .05, the cores .5″ dia., 3″ long, and 2″ apart, center to center, and the M.M.F. 4,000?

SOLUTION. — $\frac{b}{d_c} = \frac{2}{.5} = 4.$ From table, when $\frac{b}{d_c} = 4$, reluctance per inch is .258, and the reluctance for 3″ is

$$\frac{.258}{3} = .086.$$

If there was no leakage the total useful lines would equal the total flux, which is

$$\frac{4,000}{.05} = 80,000,$$

but the leakage is,

$$\frac{\left(\frac{4,000}{2}\right)}{.086} = 23,250;$$

therefore the useful lines are

$$80,000 - 23,250 = 56,750,$$

and the leakage coefficient

$$V_l = \frac{\phi}{\phi_1} \ (37) \ = \frac{80,000}{56,750} = 1.41. \quad \textit{Ans.}$$

Therefore, the total reluctance may be said to be $.05 \times 1.41 = .0705$ including the leakage.

Also the M.M.F. must be increased approximately 1.41 times to produce 80,000 useful lines through the poles and armature of the magnet.

A high reluctance in the cores complicates the problem, as the M.M.F. between the poles can not be considered as the total M.M.F.

In general, the leakage may be reduced by the uniform distribution of the winding over the magnetic circuit, roundness and evenness of the magnetic circuit, and the avoidance of sharp corners and abrupt turns.

21. Limits of Magnetization.

When the magnetizing force about an iron or steel core is gradually increased from zero, the magnetization in the iron also increases, though not in the same proportion,

until it reaches a point where it is not affected by a very material increase in the magnetizing force. The iron is then said to be *saturated*, and the point at which the induction reaches the maximum is called the *saturation point*, or the *limit of magnetization*.

The following table * shows the various values of B for different grades of iron and steel at the saturation point.

	Values of B
Wrought iron	130,000
Cast steel	127,500
Mitis iron	122,500
Ordinary cast iron	77,500

The practical working densities are about two-thirds of the densities given in the above table. For practical working densities see table, p. 117.

The relation between the values of H and B can be plotted as a curve which has the general form as in Fig. 22. (Also see Figs. 18 and 19.)

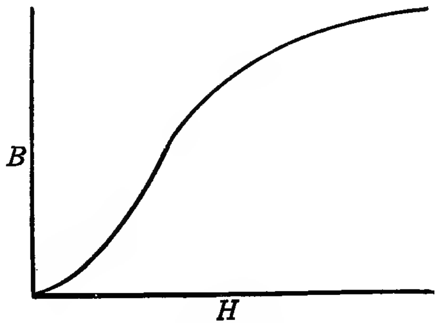


Fig. 22.

* Wiener, Dynamo-Elec. Machinery.

22. Hysteresis.

If now the magnetizing force is gradually reduced from the maximum value to zero, the magnetization will be found to have a higher value in the decreasing series of $\mathcal{H}$ than in the increasing series.

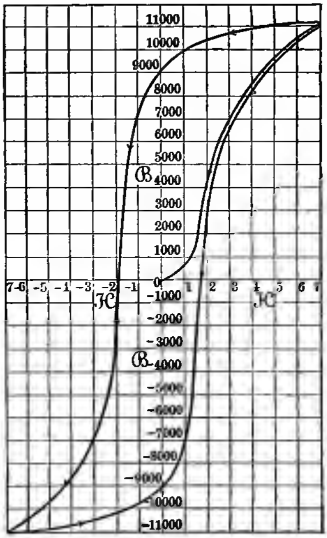


Fig. 23.

Consulting the curve, Fig. 23, in order to bring $\mathcal{B}$ down to zero, it is necessary to make $\mathcal{H}$ almost -2; that is, the magnetizing force has to be reversed and brought to a considerable negative value before the magnetism reaches zero.

Continuing the process, $\mathcal{B} = -11{,}000$ when $\mathcal{H} = -7$, and if now $\mathcal{H}$ is brought to zero, and then made positive

again, the corresponding values of $\mathfrak{B}$ will be found on the curve at the right.

This diagram is taken from one of Ewing's tests of a very soft iron ring, and is known as the *Hysteresis Loop.*

The area of the loop will vary for different grades of iron or steel, but the general form will always be the same, and the values of $\mathfrak{B}$ in the increasing series will never be quite equal to the decreasing value.

The area of the loop represents the loss due to heat in the iron, and is called the *Hysteresis Loss*, which in alternating-current apparatus is very serious. This energy loss is expressed by the following formula due to Steinmetz:

$$W_c = f n_c \mathfrak{B}^{1.6}, \tag{41}$$

where W_c = watts lost per cubic centimeter of iron,
f = number of cycles per second,
$\mathfrak{B}$ = maximum induction per square centimeter,
n_c = a constant varying from .002 for soft irons to .0045 for transformer irons.

23. Retentiveness.

That property which tends to retain magnetization is known as *Retentiveness*, and that portion of magnetization which remains is called *Residual Magnetization*, and the force which maintains the magnetization is called the *Coercive Force.*

The reason why rapid-acting electromagnets or induction coils have openings in the magnetic circuit is because the iron makes the action sluggish due to the retentiveness. If the armature of an electromagnet actually touches the pole pieces, it will stick after the current ceases to flow through the winding.

Problems.

1. The E.M.F. of an electric circuit is 110 volts and the resistance is 5 ohms. How many amperes of current will flow through the circuit? 22 amperes.

2. The current strength of a circuit is 20 amperes and the E.M.F. is 220 volts. What is the resistance of the circuit? 11 ohms.

3. A coil of wire has a resistance of 100 ohms. What will be the E.M.F. across its terminals when a current of .5 ampere is flowing through the winding? 50 volts.

4. In Problem 3, how many watts would be expended on the winding? 25 watts.

5. What would be the expenditure in watts in Problem 1? 2,420 watts.

6. How many watts would be expended in Problem 2? 4,400 watts.

7. How many amperes would be required to produce 5 horse-power in a circuit at 110 volts? 33.91 amperes.

8. What would be the resistance of the circuit in Problem 7? 3.24 ohms.

9. How many horse-power in the circuit in Problem 1? 3.24 H.P.

10. Two coils of 25 ohms and 50 ohms respectively are connected in series in a 10-volt circuit. How many watts will they consume, assuming the resistance of the coils to be the entire resistance of the circuit? 1.33 watts.

11. In Problem 10, how many watts would be consumed if the two coils were connected in parallel? 6 watts.

12. Three coils of 5 ohms, 12 ohms, and 17 ohms respectively are connected in multiple. What is the joint resistance? 2.92 ohms.

13. In Fig. 7, what would be the voltage across the terminals of ρ_4, if $\rho_1 = 4$, $\rho_2 = 7$, $\rho_3 = 11$, and $\rho_4 = 3$? 1.7 volts.

14. The M.M.F. of a magnetic circuit is 1,200 gilberts and the reluctance is .001 oersted. How many webers will flow through the circuit? 1,200,000 webers.

15. A flux of 10,000 webers is obtained with 1,500 gilberts. What is the reluctance? .15 oersted.

16. How many gilberts will be required to force 20,000 webers through a reluctance of .025 oersted? 500 gilberts.

17. What is the reluctance of a magnetic circuit 10 centimeters in length and 1 square centimeter in cross-section, if the permeability is 1,800? .00555 + oersted.

18. How many gilberts would be required to force 100 webers through an air gap .1 centimeter long by 2 square centimeters in cross-section? 5 gilberts.

19. What is the M.M.F. in 2 centimeters of length measured along one line of force? 2 gilberts.

20. What is the permeability of an iron core in which the induction $\boldsymbol{B}$ is 6,200, with a magnetizing force of $H = 4$? $\mu = 1{,}550$.

21. What intensity will be required for an induction $\boldsymbol{B}$ of 5,000 gausses when $\mu = 1{,}500$? $H = 3.33$.

22. How many ampere-turns would be required to force 16,000 lines of force through a magnetic circuit 10 inches long and 2 square inches in cross-section, and with permeability $\mu = 2{,}100$? 11.93 ampere-turns.

23. What would be the total flux produced by 200 ampere-turns in a magnetic circuit 16 inches long, 1.5 square inches in cross-section, and with permeability μ = 2,000? 119,738 webers.

24. In Problem 23, what would be the intensity of induction B? $B = 79{,}825$.

25. How many ampere-turns would be required to produce a density of magnetization B of 55,000 lines per square inch if $l = 8$ and $\mu = 1{,}700$? 81.06 ampere-turns.

26. What would be the density of magnetization B when $IN = 200$, $l = 11$, and $\mu = 1{,}300$? $B = 75{,}471$.

27. What is the magnetizing force when $l = 7$ and $IN = 3{,}000$? $H = 1{,}368$.

28. In 25, what are the ampere-turns required per linear inch of magnetic circuit? 10.13 ampere-turns.

29. What is the leakage coefficient where the useful flux is 120,000 lines and the total flux is 180,000 lines? $V_1 = 1.5$.

30. What is the useful flux when the total flux is 90,000 lines and the leakage coefficient 1.4? $\phi_1 = 64{,}290$.

31. What is the air reluctance between the cores of a magnet each 1″ diameter, 3″ long, and 2″ apart, center to center? $R = .055$.

32. How many ampere-turns would be required in Fig. 20 to force 50,000 lines per square inch through the cores when the armature is removed $\frac{1}{16}$″ from the poles, not considering leakage? Approximately 2,000 ampere-turns.

CHAPTER II.

WINDING CALCULATIONS.

24. Simple Principle of Calculating Windings.

THE length of any strand which may be wound in any given bobbin of any shape or form depends upon two things only, viz., the available volume of the bobbin, and the cross-sectional constant.

Let l_w = length of strand in inches,
V = volume of winding space in square inches,
g^2 = cross-sectional constant.

Then, $l_w = \frac{V}{g^2}$ (42), $g^2 = \frac{V}{l_w}$ (43), $V = g^2 l_w$ (44).

The cross-sectional constant must not be confused with the cross-section of the strand. To make the meaning clear, assume that a strand of round insulated wire is wound in two layers on a tube, as in Fig. 24, shown in cross-section.

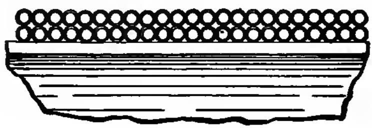


Fig. 24.

It will be seen at once that the cross-sectional area of the insulated wire is $g^2 \times .7854$, in which g represents the diameter of the insulated wire, while the actual area

consumed is equal to g^2, which is the area of each square. While this is only approximately correct on account of the imbedding of the wires, it illustrates the general principle. As a matter of fact, when winding with insulated wire, it is best to fill a known volume with the wire, noting the length of the wire, and then working backwards by using formula (43), $g^2 = \frac{V}{l_w}$. Then, with the known value of g^2, a volume may be calculated to suit any required length of wire, or the length of wire may be calculated which will just fill a given bobbin.

As the resistance of an electrical conductor of constant cross-section varies directly with its length, it is evident that the resistance of any wire which may be contained in any bobbin or winding volume may be readily calculated.

25. Copper Constants.

Now, it has been found by careful experiment that a commercial soft drawn copper wire, .001″ in diameter, has a resistance of 10.3541 ohms per foot at 68° F. Therefore, to find the resistance in ohms per foot for any other copper wire, divide 10.3541 by the area of the wire in *circular mils*, a *mil* being one-thousandth of an inch, and a circular mil the square of the diameter in mils. Thus, a wire .005″ (5 mils) in diameter has a cross-sectional area of 25 circular mils, expressed 25 C.M.

Thus, ohms per foot $$\omega' = \frac{10.3541}{\text{C.M.}}. \qquad (45)$$

Now suppose it is required to know what resistance ρ may be obtained in a given bobbin of volume V, with in-

sulated copper wire of diameter Δ. Assume that the available winding volume $V = 1.68$ cu. in., and the diameter of the wire is .010″ (10 mils), and that it is covered with cotton insulation which brings the total diameter up to .014″ $= g$, since

$$g = \Delta + i. \tag{46}$$

Then, by using formula (42),

$$l_w = \frac{V}{g^2} = \frac{1.68}{.014^2} = \frac{1.68}{.000196} = 8{,}570'' = 715 \text{ feet.}$$

By formula (45),

$$\text{ohms per foot} = \frac{10.3541}{\text{C.M.}} = \frac{10.3541}{10^2} = \frac{10.3541}{100} = .103541.$$

The resistance ρ is equal to the product of the ohms per foot times the number of feet.

$$\therefore \rho = .103541 \times 715 = 74 \text{ ohms.}$$

The foregoing is given merely to make the reader thoroughly familiar with the underlying principle. The methods of calculating volumes of various forms of bobbins will be given in subsequent pages.

26. Most Efficient Winding.

The most efficient winding for developing or absorbing magnetic energy is one in which the resistance is low and the turns are numerous, since the magnetomotive force is proportional to the ampere-turns. Since the current is equal to the voltage divided by the resistance, the lower the resistance the greater will be the current flowing through the turns of wire, thereby increasing the ampere-turns, and where a constant resistance is required, if the

number of turns may be increased, the ampere-turns will increase also, in direct proportion to the number of turns.

For this reason, the wire with lowest resistance should be used; and as copper fulfills the practical requirements, all wires will be understood to be of copper unless otherwise specified.

The ampere-turns depend upon two things only, viz., the voltage and the resistance of the average turn. To make the meaning clear, assume the resistance of the mean or average turn to be one ohm, and the voltage 100 volts. According to Ohm's law, the current in amperes would be

$$I = \frac{E}{\rho} = \frac{100}{1} = 100 \text{ amperes.}$$

100 amperes and 1 turn = 100 ampere-turns $= IN$. Now assume 10 turns of wire instead of 1 turn. The resistance will increase directly with the number of turns, therefore the resistance would be 10 ohms and the current 10 amperes, and consequently there would be 100 ampere-turns as before.

In calculating the above, the average turn must always be taken, for the resistance of the turns increases directly as the diameter increases.

27. Circular Windings.

A round core is the most economical form, as more turns of wire may be wound thereon with a given amount of copper, for the same cross-section of core. Since the leakage from core to core, for equal mean distances apart, is proportional to the surface of the core, the round core

has a decided advantage, as it has the minimum surface for equal sectional areas, and there are no sharp edges to facilitate leakage, therefore it is very commonly used.

The winding on a round core is really a hollow cylinder, and its volume is equal to πMLT.

Where M = the average diameter of the winding,
L = length of winding,
T = thickness of winding,
π = 3.1416 = ratio between diameter and circumference of circle.

(See Fig. 25.)

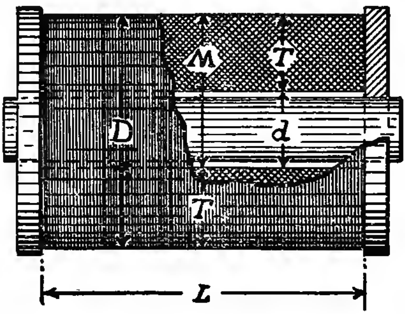


Fig. 25.

$$M = \frac{D + d}{2}. \qquad (47) \qquad T = \frac{D - d}{2}. \qquad (48)$$

Where D = true diameter of winding,
d = diameter of core + sleeve.

$$\therefore\ MT = \frac{D^2 - d^2}{4}, \qquad (49)$$

and the volume

$$V = \pi L \left(\frac{D^2 - d^2}{4}\right); \qquad (50)$$

$$\text{also}\quad M \overset{(51)}{=} D - T \overset{(52)}{=} T + d, \quad T \overset{(53)}{=} D - M \overset{(54)}{=} M - d,$$

$$D \overset{(55)}{=} 2T + d \overset{(56)}{=} M + T, \quad d \overset{(57)}{=} D - 2T \overset{(58)}{=} M - T.$$

Since the unit of length throughout these calculations will be the inch, the resistance of conductors must be reduced to *ohms per inch*, and *circular mils* to *circular inches*, one circular inch being 1,000,000 circular mils. Thus, the cross-sectional area of a wire .001″ in diameter is one circular mil or .000001 circular inch.

Formula (45) then reduces to —ohms per inch,

$$\omega'' = \frac{10.3541}{12\ \text{C.M.}} = \frac{.86284}{\text{C.M.}}. \tag{59}$$

Substituting circular inches for circular mils,

$$\omega'' = \frac{.00000086284}{\Delta^2}. \tag{60}$$

Substituting the value of V in (42),

$$l_w = \frac{\pi L (D^2 - d^2)}{4 g^2}. \tag{61}$$

Since the resistance ρ is equal to the product of the ohms per inch times the number of inches,

$$\rho = \frac{.00000086284\,\pi L (D^2 - d^2)}{4 g^2 \Delta^2}$$
$$= \frac{.0000027107\ L (D^2 - d^2)}{4 g^2 \Delta^2}. \tag{62}$$

Assigning c to .0000027107, and K to $\pi\omega''$,

$$K = \frac{c}{\Delta^2}, \tag{63}$$

formula (62) reduces to

$$\rho = \frac{KL(D^2 - d^2)}{4g^2}. \tag{64}$$

Let
$$R = \frac{K}{g^2}, \tag{65}$$

then
$$\rho = \frac{RL(D^2 - d^2)}{4} \ (66) = RMLT. \tag{67}$$

Hence R is the resistance which will be obtained when $MLT = 1$, and is the ratio between winding volume and resistance.

Therefore, K is the resistance factor, g^2 is the space factor, and R is the combined resistance and space factor.

From (67) it follows that

$$R = \frac{\rho}{MLT} \ (68) = \frac{4\rho}{L(D^2 - d^2)}. \tag{69}$$

The tables on pp. 137–147 are calculated on this principle, and the proper wire to be used for the required resistance is found opposite the value of R.

As in practice the value of R usually falls between two sizes of wire, the smaller wire is taken and a new value found for D by the formula deduced from (69).

$$D = \sqrt{\frac{4\rho}{RL} + d^2}. \tag{70}$$

This gives the actual theoretical diameter of the winding when a standard insulated wire is used.

Example. — Given $D = 1$. $d = .43$. $L = 2$.

(*a*) What size of single silk-covered wire must be used so that a resistance of 500 ohms may be obtained?

(*b*) What will be the actual value of D?

SOLUTION. — From (69),

$$R = \frac{4\rho}{L(D^2 - d^2)} = \frac{2,000}{2(1 - .185)} = \frac{2,000}{1.63} = 1,228.$$

In the table, the nearest R value for silk-covered wire is 1,480 opposite No. 35 wire. *Ans.* (*a*).

The actual diameter of the winding will be

$$D = \sqrt{\frac{4\rho}{RL} + d^2} \qquad (70)$$

$$= \sqrt{\frac{2,000}{2,960} + .185} \doteq \sqrt{.676 + .185}$$

$$= \sqrt{.861} = .928. \quad \textit{Ans. } (b).$$

To find the internal diameter of the winding, use formula derived from (70),

$$d = \sqrt{D^2 - \frac{4\rho}{RL}}. \qquad (71)$$

As previously stated, the factor R may be better understood as a combined space and resistance factor. The resistance of a conductor varies inversely as the square of its diameter, and the length of a conductor that will fill a given winding volume varies inversely as the square of its diameter also. Therefore, as a combined space and resistance factor, R varies inversely as $\Delta^2 \times \Delta^2 = \Delta^4$. Hence, $R = \frac{c}{\Delta^4}$, not considering insulation.

When insulation is considered, as it always should be in practice,

$$R = \frac{c}{\Delta^2(\Delta + i)^2}. \qquad (72)$$

The following is a simple method of finding MT, as the diameters do not have to be squared:

$$T = \frac{D-d}{2} \text{ (48), and } M = T + d \text{ (52).}$$

Thus,

$$\begin{array}{rl} 1.00 & = D \\ .36 & = d \\ \hline 2)\ .64 & \\ .32 & = T \\ .36 & = d \\ \hline .68 & = M \end{array} \qquad \therefore\ MT = .32 \times .68 = .2176.$$

The length of wire in inches in any winding is

$$l_w = \frac{\pi MLT}{g^2}. \tag{73}$$

28. Points to be Observed in Practice.

In calculating magnet windings, the utmost care must be exercised in measuring the exact dimensions of the winding volume, otherwise the results will vary greatly.

When it is considered that the resistance varies inversely with the square of the diameter of the wire plus insulation, it is readily seen that these factors are most important. The variation of one-thousandth of an inch in the thickness of the insulation on a No. 35 wire will cause a variation of 23 per cent in the amount of wire that may be wound in a given winding volume.

Also, the diameter of the core to wind on must be taken, i.e., the diameter after the paper or mica or other insulating material has been wrapped around the core, for the thickness of the insulating sleeve, though slight, makes a great difference in the volume of the winding. The true outside diameter must also be accurately measured for the same reason as with the core, but with the outside diameter the variation is even more marked.

In the case of the bobbin in Fig. 26, if the winding was calculated to contain 950 ohms of No. 36 single silk-covered wire, using the actual dimensions of the bobbin

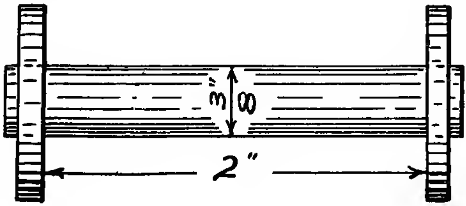


Fig. 26.

without considering the insulating sleeve, and then it was wound apparently full of wire but really to but .95 inch diameter, the resistance would be but 795 ohms, or a difference of 19.5 per cent.

Again, if the above wire was stretched during the winding operation so that the diameter was reduced from .005″ to .0049″, a difference almost too small to detect with a micrometer, there would be an increase in resistance of 7 per cent.

Assuming that the insulation on the wire is 3 mil increase, there will be another error of 4.6 per cent in the weight of the wire in the winding.

Thus it is seen that too much precaution cannot be observed.

The proper way to measure the outside diameter of a winding is to measure the length of a piece of paper which will just go around it, and then dividing it by π.

Thus, $D = \frac{l_p}{\pi}$ (74), where l_p = length of wrap of paper.

If measured with calipers the winding will be found to be slightly elliptical, rendering it impossible to measure the exact diameter accurately.

29. Formulæ for Turns, Resistance, and Ampere-turns.

The number of turns which may be contained in any bobbin of any form is equal to one-half the longitudinal-sectional area divided by g^2; thus,

$$N = \frac{TL}{g^2}. \tag{75}$$

The resistance may be easily found when the turns are known, by multiplying the number of turns by the resistance of the average turn, or

$$\rho = \pi\omega'' MN \tag{76}$$
$$= KMN. \tag{77}$$

To find the insulated wire and resistance when dimensions of the bobbin and the number of turns are given, proceed as follows:

Find the value of g^2 by formula deduced from (75).

$$g^2 = \frac{TL}{N}; \tag{78}$$

then from the table select the next smaller size for g^2 and calculate the new diameter for this value of g^2 by the formula

$$D = \frac{2Ng^2}{L} + d, \tag{79}$$

and with the new value of D the resistance is calculated by formula (67),

$$\rho = \frac{RL(D^2 - d^2)}{4}.$$

Ampere-turns are calculated by the formula

$$IN = \frac{E}{KM}, \tag{80}$$

KM representing the resistance of the average turn. In this it is seen that the length of the winding makes no difference in the number of ampere-turns, for as the length and turns increase, the resistance increases and the current decreases in the same ratio, thus keeping the ampere-turns constant for constant voltage. With constant current, however, the ampere-turns and voltage would increase directly with the turns and resistance.

The length of the bobbin affects the heating of the winding, and will be explained fully farther on.

In order to find the resistance of a coil of wire having a definite number of ampere-turns with a certain size of wire, it is necessary to find the outside diameter of the winding. This is found by the following formula derived from (80):

$$D = \frac{2E}{KIN} - d. \tag{81}$$

To find the exact diameter of wire to produce a certain number of ampere-turns in a given winding space with given voltage, use formula derived from (63) and (80).

$$K = \frac{c}{\Delta^2}. \quad (63) \qquad\qquad IN = \frac{E}{KM}. \quad (80)$$

$$\therefore \Delta = \sqrt{\frac{cINM}{E}}. \tag{82}$$

From (75), $N = \dfrac{TL}{g^2}$. $\therefore T = \dfrac{Ng^2}{L}$. (83)

Thus, when given turns, inside diameter, length of winding, and size of wire and insulation, to find the outside diameter, resistance, and weight, proceed as follows:

$$T = \frac{Ng^2}{L}. \quad (83) \qquad D = \frac{2\,Ng^2}{L} + d. \quad (79)$$

$$M = T + d \; (52) = \frac{Ng^2}{L} + d. \tag{84}$$

$$\therefore MLT = L\left[\frac{Ng^2}{L}\left(\frac{Ng^2}{L} + d\right)\right] \tag{85}$$

$$= Ng^2\left(\frac{Ng^2}{L} + d\right), \tag{86}$$

or

$$MLT = \frac{Ng^2\,(Ng^2 + dL)}{L}. \tag{87}$$

Therefore, the MLT is found directly by Formula (87); the resistance or weight of the winding is then found directly by substituting the values MLT and multiplying by R and w, as in formulæ (88) and (89).

$$\rho = \frac{RNg^2\,(Ng^2 + dL)}{L} \tag{88}$$

$$Lb = \frac{wNg^2\,(Ng^2 + dL)}{L} \tag{89}$$

where Lb = weight in pounds.

w is the combined resistance and space factor, and bears the same relation to weight that R bears to resistance. Thus, $Lb = wMLT$. (90)

$$w = \frac{R}{\theta} \; (91) = \frac{K}{\theta g^2} \; (92)$$

when θ = ohms per pound for insulated copper wires.

30. Constant Resistance with Variable Insulation.

When the resistance of a bobbin is constant with a given size of wire, the outside diameter of the winding will be increased by an increase in the thickness of the insulation, but the number of turns will decrease as the

thickness of the winding increases, because the length of the average turn increases with the diameter.

In this case all dimensions of the bobbin are constant excepting D. The diameter and resistance of the wire is constant, but the thickness of the insulation changes.

The number of turns for any thickness of insulation will be

$$N = L\left[\frac{\sqrt{\frac{4\rho g^2}{KL} + d^2} - d}{2g^2}\right]. \tag{93}$$

As an illustration, assume a bobbin with $d = .43$ and $L = 2$, wire No. 36, and resistance 500 ohms. If the insulation on the wire brought the total diameter g up to .008″, the turns would be 7,030. If $g = .009$, the turns would be 6,600, or a loss of over 6% for the same resistance, but with a difference of .001″ in the value of g.

31. Layers and Turns Per Inch.

To find the number of layers of wire that may be wound in thickness of winding T, use formula

$$n = \frac{T}{g_v}. \tag{94}$$

The number of turns of wire per inch,

$$m = \frac{1}{g_l}. \tag{95}$$

Thus,

$$N = mnL, \tag{96}$$

$$n = \frac{N}{mL}, \tag{97}$$

$$m = \frac{N}{nL}, \tag{98}$$

$$L = \frac{N}{mn}, \tag{99}$$

$$g_l = \frac{1}{m}, \tag{100}$$

$$g_v = \frac{T}{n}. \tag{101}$$

Where $g_v =$ vertical value of wire and insulation,
$g_l =$ lateral value of wire and insulation.

The above formulæ are often found convenient in practice.

32. Windings with Wires Other than Copper.

To find the resistance of a winding with wire of different specific resistance than copper, multiply the resistance of copper by the coefficent for the other kind of wire. Thus, 18% German silver wire has a specific resistance 18.76 times that of copper; hence the ohms per foot equal

$$\frac{10.3541 \times 18.76}{\Delta^2} = \frac{194}{\Delta^2}.$$

The simplest method is to find what resistance would be obtained in the given bobbin if copper wire were used, and then multiplying by the coefficient for the other wire.

33. Small Magnets on High-Voltage Circuits.

In practice it is often required to place a small electromagnet or solenoid in a high-voltage circuit, and hence the resistance of the winding must be very great in order that the winding may not be overheated. This resistance is often too high to be obtained in the given winding volume with the finest insulated copper wire on the market. It would be impracticable to use a wire of greater

resistivity than copper, as the turns, and consequently the ampere-turns, would fall short of requirements; therefore, the usual method is to wind part of the magnet with copper wire, and the balance with a high-resistance wire, using enough of the copper to give the greatest number of turns, and still leave enough room for the high-resistance wire winding to be wound over the copper wire winding.

Since the resistance

$$\rho = RL\left(\frac{D^2 - d^2}{4}\right), (66) \quad \frac{4\rho}{L} = R(D^2 - d^2).$$

If we now represent the outside diameter of the copper winding, and consequently the internal diameter of the high-resistance wire winding by X, and let ρ equal the total resistance and the R value for the high-resistance wire winding by R_1, the formula becomes

$$\frac{4\rho}{RL} = R(X^2 - d^2) + R_1(D^2 - X^2).$$

$$\frac{4\rho}{L} = RX^2 - Rd^2 + R_1 d^2 - R_1 X^2.$$

$$RX^2 - R_1 X^2 = \frac{4\rho}{L} + Rd^2 - R_1 D^2.$$

$$X^2 = \frac{\frac{4\rho}{L} + Rd^2 - R_1 D^2}{R - R_1}.$$

$$\therefore X = \sqrt{\frac{\frac{4\rho}{L} + Rd^2 - R_1 D^2}{R - R_1}}.$$

Changing signs,

$$X = \sqrt{\frac{R_1 D^2 - \left(\frac{4\rho}{L} + Rd^2\right)}{R_1 - R}}. \qquad (102)$$

EXAMPLE. — A bobbin with dimensions $D = 1$, $d = .43$, $L = 1.5$, is to contain 10,000 ohms, with the greatest amount of No. 40 copper wire, and just enough No. 40 30% German silver wire to give the resistance within the limiting O.D. (outside diameter).

SOLUTION. —

R for No. 40 S.S.C. copper wire = 12,690.
R for No. 40 – 30% – S.S.C. German silver wire = 12,690 × 28.1 = 356,590.

$$X = \sqrt{\frac{356{,}590 - \left(\frac{40{,}000}{1.5} + 2{,}348\right)}{356{,}590 - 12{,}690}}$$

$$= \sqrt{\frac{356{,}590 - 29{,}015}{343{,}900}} = \sqrt{.9525} = .976. \quad \textit{Ans.}$$

Therefore, wind to a diameter of .976″ with copper wire, and there will be just enough room left for the German silver wire to bring the total resistance up to 10,000 ohms.

34. Resistance Wires.

The resistance wires most commonly used in connection with electromagnet windings to increase the resistance so that they may be used on a circuit of high voltage, are German silver and Climax.

The commercial German silver wires are of two grades; viz., 18% and 30% nickel. The 18% wire has about 18.8 times the resistance of copper, and the 30% wire 28.1 times the resistance of copper.

The temperature coefficient for German silver wire is .00017 per degree F. for 18%, and .0001185 per degree

F. for 30%. Specific gravity approximately 8.5. Climax wire has about 48 times the resistance of copper. Its temperature coefficient is .00042 per degree F. and its specific gravity 8.137.

Under similar conditions, the carrying capacity of two wires of equal diameter but of different materials varies inversely as the square root of their specific resistances.

The German silver wire table on p. 148 is based on tests made by a well-known manufacturer; the ohms per pound are based on an average specific gravity of 8.5 for both 18% and 30%.

As the resistance of German silver varies in the same specimen, the table is to be considered as commercially correct, but not absolutely correct.

35. One Coil Wound Directly Over the Other.

Simply figure for the total number of turns, using formula (79),

$$D = \frac{2Ng^2}{L} + d.$$

The total resistance can then be calculated by formula (66).

$$\rho = \frac{RL(D^2 - d^2)}{4}.$$

This is not absolutely correct, as paper is usually wrapped between the two windings. Therefore, the resistance of each coil may be calculated, using the D of the first coil plus paper, for the d of the outer coil.

The latter method is necessary when different sizes of wires are used on successive windings.

36. Parallel Windings.

In certain types of electromagnetic apparatus, especially those used in telephone and telegraph work, the coils consist of two parallel wires coiled simultaneously, each being insulated from the other, thus forming two separate and distinct circuits.

In telegraph work the coils thus wound are used on polarized relays, and either act separately, in conjunction with each other, or in opposition to each other; thus, when the current is flowing through one coil only, it acts with a certain strength, and when the same amount of current flows through both coils *in the same direction*, the magnet will have approximately twice the strength.

Again, if the same strength of current flows through both coils simultaneously, but *in opposite directions*, there will be no magnetic action whatever, as the magnetic effect of one coil is neutralized by the effect of the other.

The latter principle is employed in the making of non-inductive resistance coils, only in this case the same current passes through both coils, thus really forming but one coil.

The differential relay is also used in telephone switchboards, but the winding referred to above for telephone work is classed under the heading of *Repeating Coils*. The telephone repeating coil is really an induction coil, but the fact of having the two windings in parallel gives it the condenser effect also, so that the inductive effect is electrostatic as well as electromagnetic.

To find the resistance of each wire in parallel windings, calculate the resistance for a single insulated wire,

and divide by 2, since it is really but one complete coil with half the resistance in each branch.

A good method of finding the respective resistances of two or more wires wound simultaneously and in parallel is to calculate the number of turns, and then apply formula (77) $\rho = KMN$, to each wire separately.

Another form of winding which may be classed under *Parallel Windings* is where two or more insulated wires are wound in parallel and the respective ends electrically connected together so as to act as one conductor. This is sometimes necessary where a wire of large cross-section is to be wound upon a very small core, as several finer ones will have the same cross-section and still be flexible.

Referring to Fig. 8 and Fig. 9, p. 10, the resistance in two wires in series is just 4 times their resistance when connected in multiple, or the resistance of any number of wires connected in series is equal to the square of the number of wires times the resistance in multiple, or

$$\rho_s = n_w^2 Jr. \tag{103}$$

Likewise, the resistance of any number of wires in multiple is equal to the resistance of the wires in series divided by the square of the number of wires, or

$$Jr = \frac{\rho_s}{n_w^2}. \tag{104}$$

The total resistance of the wire used is of course equal to the resistance of all the wires connected in series.

Likewise, the resistance of each wire is equal to the total resistance divided by the number of wires.

37. Joint Resistance of Parallel Windings.

To find the joint resistance of any number of wires of equal resistance in a coil, divide the resistance of each wire by the number of wires in the coil. Thus,

$$Jr = \frac{\rho}{n_w}. \qquad (105)$$

To find the resistance of each wire, multiply the joint resistance by the number of wires,

$$\rho = Jrn_w. \qquad (106)$$

The total resistance is equal to the joint resistance multiplied by the square of the number of wires,

$$\rho_s = n_w^2 Jr. \qquad (103)$$

38. Relations Holding for Any Size of Wire and Winding Volume.

If the ratio between wire and insulation was constant for all sizes of wire, the following laws would hold for a given winding volume:

1. For any size of wire, the length of wire, and consequently the number of turns, varies as the cross-section of the wire.

2. The resistance varies as the square of the number of turns, or inversely as the square of the cross-section of the wire, or inversely as the fourth power of the diameter of the wire.

3. The current at any given voltage varies inversely as the square of the number of turns, or inversely as the resistance.

4. The weight of wire is constant for any size.

5. The magnetic effect varies as the current multiplied by the square root of the resistance, or as the square of the diameter of the wire.

In practice, however, there is a wide variation between large and small insulated wires, as may be seen by consulting the insulated wire tables.

For every two sizes of insulated wire the resistance is approximately doubled, and for each consecutive size it is half as much again; or, in other words, the resistance increases approximately 50% for each consecutive size of insulated wire with the same insulation, and increases approximately 25% for half-sizes.

39. The American Wire Gauge. (B. & S.)

Wire gauges are arranged in the form of a geometrical series.

The sizes are determined by the diameter of the circles, which may be placed between two lines at a given angle, in such a manner that the circles will just touch one another and the bounding lines, as in Fig. 27.

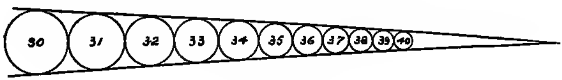


Fig. 27.

The American wire gauge is based on the geometrical series in which No. 0000 is .46″ diameter, and No. 36 is .005″ diameter. The angle of divergence is 6°–36′–40″.*

* W. J. Varley.

As there are just 39 sizes between No. 0000 and No. 36, the factorial difference (or progression ratio) between any two sizes is

$$\sqrt[39]{\frac{.46}{.005}} = \sqrt[39]{92} = 1.122932.$$

Thus, to find the diameter of any wire,

$$\Delta = \frac{.46}{1.122932^{(s+3)}}, \qquad (107)$$

where $\qquad s =$ the desired gauge number.

This is most conveniently worked out by means of logarithms.

$$\text{Log } .46 = \bar{1}.6627578.$$
$$\text{Log } 1.122932 = .0503535.$$
$$\therefore \log \Delta = \log .46 - [(s+3) \log 1.122932] \qquad (108)$$
$$= \bar{1}.6627578 - [(s+3)\,.0503535]. \qquad (109)$$

Example. — What is the diameter of No. 30 wire?

Solution. —

$$\text{Log } \Delta = \bar{1}.6627578 - (33 \times .0503535).$$
$$\therefore \text{Log } \Delta = \bar{1}.6627578 - 1.6616655 = \bar{2}.0010923.$$
$$\therefore \qquad \Delta = .0100252. \quad \textit{Ans.}$$

The exact diameter of half or quarter sizes may be found by the same rule.

To find the gauge number corresponding to any diameter of wire, proceed as follows:

$$\text{Since } \log \Delta = \bar{1}.6627578 - [(s+3)\,.0503535]. \qquad (109)$$
$$s = \frac{\bar{1}.6627578 - \log \Delta}{.0503535} - 3. \qquad (110)$$

EXAMPLE. — What size in the American Wire Gauge is a wire .0085″ in diameter?

SOLUTION. —

$$s = \frac{\bar{1}.6627578 - \bar{3}.9294189}{.0503535} - 3$$

$$\therefore\ s = \frac{1.7333389}{.0503535} - 3 = 34.423 - 3 = 31.423,$$

or approximately No. 31.4. *Ans.*

In the American Wire Gauge the ratio of diameters for every six sizes is nearly 2, the exact ratio being 2.0050 +; that is, the diameter of No. 30 is twice that of No. 36; No. 18 is twice the diameter of No. 24, etc.

The cross-sectional areas of the wires vary in the ratio of nearly 2 for every three sizes. Thus, No. 37 has twice the cross-sectional area of No. 40, etc.

This is important to remember, as it is found very convenient in estimating mentally the size of wire to have a certain resistance when the resistance and length of another size of wire are known.

The resistance of a bobbin, however, will vary nearly in the ratio of 2 for every two sizes of wire, since there will be less length of heavy wire in a given volume than there would be of finer wire in the same volume. Thus, a bobbin containing 500 ohms of No. 34 wire would contain approximately 1,000 ohms of No. 36 wire.

From the data given in the "Explanation of Table," on page 136, the following relations hold:

$$\text{Pounds per foot} = 3.0269\ \Delta^2. \qquad (111)$$

$$\text{Pounds per ohm} = 292{,}400\ \Delta^4. \qquad (112)$$

$$\text{Feet per pound} = \frac{.33036}{\Delta^2}. \qquad (113)$$

$$\text{Feet per ohm} = 96{,}585\ \Delta^2. \qquad (114)$$

$$\text{Ohms per pound} = \frac{.00000342}{\Delta^4}. \qquad (115)$$

$$\text{Ohms per foot} = \frac{.0000103541}{\Delta^2}. \qquad (116)$$

40. Thickness of Insulation.

To find the thickness of insulation permissible, on a wire when the exact resistance has been calculated for heating or other conditions, proceed as follows:

From (63), $K = \frac{c}{\Delta^2}$,

and from (46), $g = \Delta + i$.

Now $R = \frac{K}{g^2}$ (65) $= \frac{c}{\Delta^2 (\Delta + i)^2}$ (72).

$$\therefore\ i = \frac{\sqrt{\frac{c}{R}} - \Delta^2}{\Delta} \qquad (117)$$

$$= \sqrt{\frac{c}{R\Delta^2}} - \Delta, \qquad (118)$$

or

$$i = \frac{\sqrt{cL(D^2 - d^2)}}{4\,\rho\Delta^2} - \Delta, \qquad (119)$$

in which the value of R from (69) has been substituted.

41. Ratio of Weight of Copper to Weight of Insulation.

The specific gravity of copper is 8.89.
The specific gravity of cotton is 1.377.
The specific gravity of silk is 1.03.

The values for cotton and silk being taken when wound tightly around the wire.

Since the weights are proportional to their specific gravities, $8.89\ \Delta^2$ is the relative weight of copper, and $1.377\ \Sigma$ is the relative weight of cotton in the insulated wire, Σ representing the cross-sectional area of the cotton.

$$\therefore\ \Sigma = g^2 - \Delta^2. \tag{120}$$

The percentage of copper in a cotton-insulated wire is

$$a = \frac{8.89\ \Delta^2}{8.89\ \Delta^2 + 1.377\ \Sigma}; \tag{121}$$

and for silk-insulated wire, per cent of copper

$$a_1 = \frac{8.89\ \Delta^2}{8.89\ \Delta^2 + 1.03\ \Sigma}. \tag{122}$$

Therefore, to find the ohms per pound for any insulated wire, first find the percentage of copper in the insulated wire, and then compare with the weight for bare wire.

Example. — What is the ohms per pound for No. 36 silk-covered wire, with two-mil increase for insulation?

Solution. —

$$\begin{aligned} g^2 &= .000049 \\ \Delta^2 &= .000025 \\ \hline \therefore\ \Sigma &= .000024 \end{aligned}$$

$$\frac{8.89\ \Delta^2}{8.89\ \Delta^2 + 1.03\ \Sigma} = \frac{.0002223}{.000247} = 90\%\ \text{copper.}$$

From the table Ω for No. 36 bare wire is 5,473.

$$\therefore\ \theta = 5{,}473 \times .90 = 4{,}926. \quad \textit{Ans.}$$

Ω = ohms per pound for bare wires.
θ = ohms per pound for insulated wires.

The weight of copper in the insulated wire may be found by multiplying the weight of the insulated wire by the percentage of copper, and then by subtracting the weight of copper from the weight of the insulated wire we get the weight of the insulation; or, since the percentage of weight of insulation is the reciprocal of the weight of copper, it may be found after the same manner as finding the weight of copper.

The tables on pp. 138 and 147 are calculated on this principle.

The above is the only sure method of computing the relative weights of copper and insulation, and the comparison of the tables so deduced, with other insulated-wire tables, will show a great discrepancy in the latter; also, that the method of assuming the weight of copper to be 90% of the gross weight is very much in error.

The percentage of weight of German silver in a cotton-insulated German silver wire will be

$$\frac{8.5\ \Delta^2}{8.5\ \Delta^2 + 1.377\ \Sigma} = \%\ \text{G.S.} \qquad (123)$$

and for silk,

$$\frac{8.5\ \Delta^2}{8.5\ \Delta^2 + 1.03\ \Sigma} = \%\ \text{G.S.} \qquad (124)$$

The ohms per pound for any grade of cotton or silk insulated wire may then be found after the manner of solving for copper wire, by consulting the German silver wire table on p. 148.

The weight of insulated wire in a winding is obviously equal to the resistance divided by the ohms per pound; therefore, the combined weight and space factor may be found by dividing the combined resistance and space factor by the ohms per pound of the insulated wire. Thus,

$$w = \frac{R}{\theta}. \qquad (91)$$

Therefore, to find the weight of wire in a winding, when the size of wire and the insulation are known, multiply the *MLT* of the winding by the combined weight and space factor w. Thus,

$$\text{Weight in pounds} = wMLT. \qquad (90)$$

42. Weight of Insulation to Insulate Any Wire.

The weight of cotton that will insulate one pound of bare copper wire is equal to

$$\frac{1.377\,\Sigma}{8.89\,\Delta^2} = \frac{.1549\,\Sigma}{\Delta^2},$$

and for silk,

$$\frac{1.03\,\Sigma}{8.89\,\Delta^2} = \frac{.1159\,\Sigma}{\Delta^2}.$$

Therefore, the weight of insulation in pounds that will insulate any weight of copper wire to a given mil increase is,

For cotton,
$$C_w = \frac{.1549\,\Sigma\lambda}{\Delta^2}, \qquad (125)$$

For silk,
$$S_w = \frac{.1159\,\Sigma\lambda}{\Delta^2}, \qquad (126)$$

where λ = weight of bare wire in pounds.

Example. — Given the size and weight of a bare copper wire, to find the weight of silk necessary to insulate it to .002″ increase.

Let $\Delta = .005 =$ No. 36 B. & S.,
$\lambda = 500.$

SOLUTION. —

From (46), $g = \Delta + i = .005 + .002 = .007.$

From (120) $\Sigma = g^2 - \Delta^2 = .000049 - .000025 = .000024.$

From (126) $S_w = \frac{.1159 \, \Sigma\lambda}{\Delta^2} = \frac{.0013908}{.000025} = 55.63$ lbs. *Ans.*

43. General Construction of Electromagnets.

The usual form of electromagnet is that shown in Fig. 28.

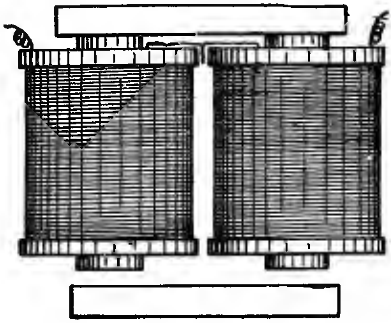


Fig. 28.

The cores, yoke, and armature are made of iron or steel, usually soft Swedish iron, for small electromagnets, and cast iron, wrought iron, or cast steel for large ones.

Cast steel is used extensively in the construction of dynamo field magnets. A magnet of cast steel is often actually cheaper as to first cost than one of cast iron, owing to the saving in weight, although it costs more per pound.

Besides the first cost is to be considered the economy of space and weight, which is very important in some forms of apparatus; also, the cost of the core is not so

important as the decrease in the cost of wire where a good quality of iron or steel is used, especially in fine wire windings.

The washers or flanges are usually made of hard rubber, fiber, or wood. Where brass spools are used, insulating material, such as paper or linen impregnated with Sterling varnish, is placed over the brass tube and against the washers.

The wire is usually insulated by winding cotton or silk about it spirally, so that adjacent turns in the winding may not come into electrical contact with one another.

Before the wire is wound on to the bobbin, the core is insulated with paper, fiber, mica, or the insulated linen mentioned above, according to the voltage to be applied to the winding.

The wire is then wound on evenly in layers by revolving the bobbin on a spindle and guiding the wire by hand. The reason why the wire should be wound evenly in layers is, that it is necessary to distribute the electrical stresses uniformly throughout the winding, thus avoiding short-circuits. If the wire is wound on carelessly, or "haphazard," as it is sometimes called, some of the first turns may lie adjacent to others which were wound on much later in the operation, thus causing a large proportion of the total voltage to exist between these turns. The result is a puncture or "breakdown" when comparatively high voltages are used on the coils.

44. Insulation of Bobbin for High Voltage.

When bobbins are made of brass, they should be thoroughly insulated with paper shellacked to the brass for

low voltages, but for high voltages special precautions must be taken.

The tube should first be covered with several wraps of the Pittsburgh Insulating Company's insulating linen, fringed at the ends; and the end flanges should also be covered with several layers of the same material, care being taken to have the slits or cuts in the linen washers at least 90 degrees apart, so that there can be no leakage at these points. It is better to assemble the linen washers before the brass washers, as then the linen washers do not have to be cut. The linen washers should be placed over the wrapping of linen on the tube, with the fringe between the metal washer and the linen washer.

If the inside terminal is to be brought out at the top of the winding, there should be several more insulating washers between the terminal and the end of the winding.

The terminals should consist of flexible rubber-covered conductor, the size varying with the size of the wire in the winding.

The coil should be thoroughly baked out and dipped in the Sterling Varnish Company's "Extra Insulating Varnish" until it is thoroughly permeated by it, and then baked until the varnish is dry.

The winding should also be covered with insulating linen and treated with Sterling varnish.

Large windings consisting of fine wire are usually covered with heavy cotton cord for mechanical protection.

Press board and Fuller board are also used for low voltages as insulating washers and covers for the winding.

45. Theory of Magnet Windings.

The winding of an electromagnet, when evenly wound in layers, consists of helices, the direction of the turns being alternately right and left; that is to say, the direction of the turns on one layer, instead of being at right angles to the core, will incline slightly to the left, whereas, in the next layer, the inclination will be to the right.

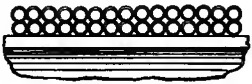


Fig. 29.

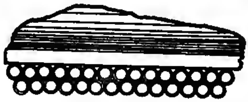


Fig. 30.

At the point where adjacent helices cross one another they appear as in Fig. 29, but diametrically opposite on the winding the turns of the upper layer sink into the groove between the turns of the layer beneath it, as in Fig. 30, then gradually leave the groove until they reach the highest point again, as in Fig. 29.

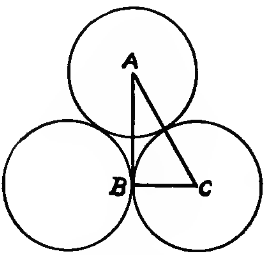


Fig. 31.

Where the imbedding occurs the following relations hold (see Fig. 31):

$$\overline{AB}^2 = \overline{AC}^2 - \overline{BC}^2. \quad \therefore AB = \sqrt{4r^2 - r^2} = r\sqrt{3},$$

where $\qquad r =$ the radius of the wire.

$$\therefore g = 2r.$$

The space consumed by the wire when imbedded is proportional to

$$2r \times r\sqrt{3} = 2r^2\sqrt{3}.$$

In the other case, however, where the wires on subsequent layers lie exactly on top of preceding layers, the space consumed is proportional to $g^2 = 4r^2$. It is therefore fair to assume that the actual space consumed in a wound magnet is proportional to the average between the two conditions, or

$$\frac{2r^2\sqrt{3} + 4r^2}{2} = 2r^2 + r^2\sqrt{3} = r^2(2 + \sqrt{3}).$$

Hence, the number of turns is proportional to

$$\frac{TL}{r^2(2 + \sqrt{3})},$$

or in terms of g^2,

$$N = \frac{TL}{\frac{g^2}{4}(2 + \sqrt{3})} = \frac{TL}{\cdot 933 g^2} = \frac{TL}{g^2} \times 1.073.$$

That is, there will be 7.3% more turns in a bobbin than if calculated on the assumption that there is no imbedding.

This, however, is somewhat counteracted by a loss at the ends, which is proportional to the turns per layer. There is a loss at the ends of one-half turn per layer.

The percentage of loss due to this is equal to the loss in turns per layer divided by the turns per layer, or

$$\text{per cent} = \frac{\cdot 5}{m}.$$

The lateral value of g is greater than the vertical value as just explained, but there is another variation due to the compression of the insulation, which has to be considered.

When the wire is wound on to the bobbin, the vertical tension is much greater than the lateral tension, and the flattening of the insulation vertically makes it spread out laterally; thus there are less turns per layer than calculated, but more layers than calculated, were this fact not taken into consideration.

However, the turns and resistance will be approximately the same as calculated.

In practice, the value of g^2 is equivalent to the square of the diameter of the wire and insulation as measured with a ratchet-stop micrometer, and the tables on pp. 138 to 147 are based on this principle.

46. Paper inserted into the Winding.

In winding a coil, and especially if fine wire is used, it is found necessary to insert stout pieces of paper occasionally between the layers to form a bridge to keep the winding smooth, otherwise little grooves appear which are due to the unevenness of the insulation, and in a short time the winding will lose all semblance of being wound in layers.

While it is almost absolutely necessary to insert this paper in the winding, it is disadvantageous, as the available

winding volume is reduced in exact ratio to the volume of paper inserted.

This may be appreciated if the paper be removed from the winding and wrapped about the core, thus forming a new d, and if the volume of the winding be calculated with this new d value, a very marked difference will be noted in the volume. Hence, only very thin, strong paper should be used, and then as sparingly as possible.

Paper inserted in the winding thus decreases the ampere-turns by increasing the outside diameter, and consequently the resistance of the average turn.

By increasing the outside diameter of the winding, however, the radiating surface is increased for the same resistance, although the increased thickness of the winding may offset this in most cases.

47. Duplex Windings.

This winding derives its name from the fact that bare wire is coiled into the winding together with a strand of silk, which insulates adjacent turns from each other laterally. The layers are insulated from each other by suitable paper. As these windings are made by automatic machinery, they are also called *Machine-wound Magnets.*

Many more turns are obtained with the same length of wire, in this form of winding, than with the common form, as the insulating materials occupy less space.

In the covered wire windings, the insulation is constant for nearly all sizes of single-covered fine wire, while the ratio of insulation to wire varies.

In the duplex winding, the ratio may be constant or variable, as desired, by the adjustment of the turns per inch.

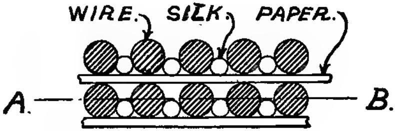


Fig. 32.

The silk lies between the wires as shown in Fig. 32; thus the wires may be much closer than if the silk lay on the common center line *AB*, and very much closer than if the silk were wrapped about the wire.

In the case of the duplex winding,

$$g_l = \Delta + S, \tag{127}$$
$$g_v = \Delta + P, \tag{128}$$
$$g^2 = (\Delta + S)(\Delta + P); \tag{129}$$

where S = silk allowance,
P = paper allowance.

It is obvious that there is no imbedding of the wires in this case, but all other relations hold as given for covered wire windings.

The winding volume consumed by the paper is

$$V_p = \pi MLPn, \tag{130}$$

and the volume consumed by the silk space is

$$V_s = \pi MLSmn. \tag{131}$$

The windings are wound in multiple on a tube of paper or other insulating material, and sawed into sections after their removal from the automatic machines. From 1 to 12

sections are wound simultaneously, according to the length of the winding.

The sections are slipped on to cores and the washers forced on to the bobbin as in the common method.

The principal features of this winding are its high efficiency and cheapness of production.

48. Other Forms of Windings than Round.

Since the round form of winding is the most common, all terms are made to apply to that type, and when other forms are used, the formulæ so arranged as to read in the same terms as those applied in the calculation of the round winding.

The other forms for which formulæ are here given are as follows:

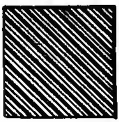


Fig. 33.

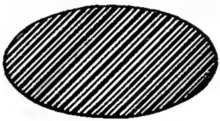


Fig. 34.

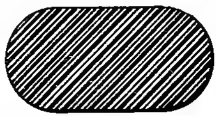


Fig. 35.

Windings on square or rectangular cores (Fig. 33), windings on elliptical cores (Fig. 34), and windings on cores, the cross-section of which is square or rectangular with rounded ends, as in Fig. 35.

It is evident that since the wire constants are fixed, all that is necessary is to express the winding volume in each case, in terms of MLT, for any form of winding.

It is to be observed also that the winding thickness T and the winding length L are constant, no matter what the form of the winding, the one point to accurately determine

being the mean perimeter factor M, which is the diameter of the mean perimeter when in the form of a circle.

This will be referred to in all cases as the mean diameter, regardless of the form of the winding.

49. Square or Rectangular Windings.

When wire is wound upon a square or rectangular core, the corners of the winding are not sharp like the corners of the core, but form arcs, the radii of which are equal to the thickness of the winding. Fig. 36 shows this principle. The four sections formed by the corners would therefore form a circle.

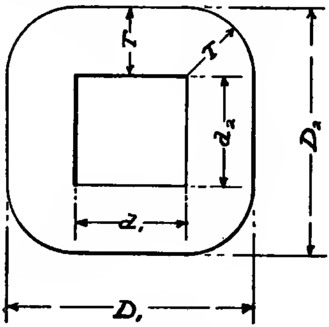


Fig. 36.

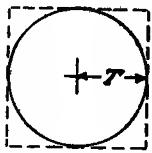


Fig. 37.

Fig. 37 shows the relative areas of the square and the circle thus formed. The area of the square with $2T$ as a side $= (2T)^2 = 4T^2$. The area of the circle with T as radius $= \pi T^2$.

Therefore, the difference between the areas of the square and the circle $= 4T^2 - \pi T^2 = .8584\, T^2$. The total end area ($A$) of the winding would therefore equal

$$(D_1 D_2 - d_1 d_2) - .8584\, T^2, \tag{132}$$

and
$$MT = \frac{A}{\pi} \tag{133}$$

$$= \frac{(D_1 D_2 - d_1 d_2) - .8584\, T^2}{\pi}. \tag{134}$$

$$\therefore\ M = \frac{(D_1 D_2 - d_1 d_2) - .8584\, T^2}{\pi T}. \tag{135}$$

$$T = \frac{D_1 - d_1}{2}. \tag{136}$$

$$T = \frac{D_2 - d_2}{2}. \tag{137}$$

From (67),
$$\rho = RMLT.$$

$$\therefore\ \rho = \frac{RL\,[(D_1 D_2 - d_1 d_2) - .8584\, T^2]}{\pi}. \tag{138}$$

$$R = \frac{\pi\rho}{L\,[(D_1 D_2 - d_1 d_2) - .8584\, T^2]}. \tag{139}$$

$$L = \frac{\pi\rho}{R\,[(D_1 D_2 - d_1 d_2) - .8584\, T^2]}. \tag{140}$$

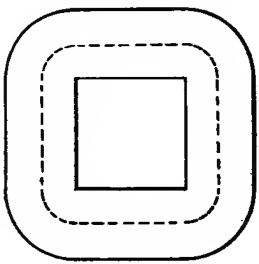


Fig. 38.

By consulting Fig. 38, it is evident that the length of the average turn (πM) for round windings $= 2\, d_1 + 2\, d_2 + \pi T$ for square or rectangular windings.

Therefore $\pi M = 2(d_1 + d_2) + \pi T$,

and $M = .637(d_1 + d_2) + T$, (141)

also $T = M - .637(d_1 + d_2)$; (142)

then $D_1 = 2T + d_1$ (143)

and $D_2 = 2T + d_2$. (144)

Since $T = \frac{D_1 - d_1}{2}$ (136),

$$M = .637(d_1 + d_2) + \left(\frac{D_1 - d_1}{2}\right); \quad (145)$$

or $M = .5 D_1 + .637 d_2 + .137 d_1$. (146)

For calculations of turns, etc., use same formulæ as applied to round windings.

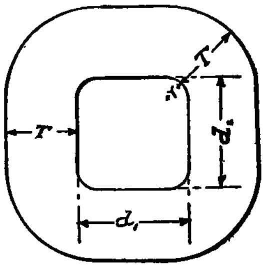


Fig. 39.

Radiating surface

$$Sr = 2L[(d_1 + d_2) + 1.5708(D_1 - d_1)]. \quad (147)$$

Substituting value of M from (135) in (80),

$$IN = \frac{E\pi T}{K[(D_1 D_2 - d_1 d_2) - .8584 T^2]}. \quad (148)$$

Or by substituting value of M from (141) in (80),

$$IN = \frac{E}{K[.637(d_1 + d_2) + T]}. \quad (149)$$

In practice, nearly all cores of square or rectangular

cross-section have more or less rounded edges, as in Fig. 39; but this need not be considered unless the radius of the arc at the edge is sufficient to make a noticeable increase in the length of the mean perimeter, πM. By inspecting Fig. 39 it will be seen that

$$M = \frac{2(d_1 - 2r) + 2(d_2 - 2r) + \pi(T + 2r)}{\pi},$$

or $\quad M = .637(d_1 - 2r) + .637(d_2 - 2r) + T + 2r.$

Clearing, $\quad M = .637(d_1 + d_2) + T - .547r; \qquad (150)$

i.e., in any case under this heading, subtract $.547r$ from the value of M in formula (141) for the true value of M.

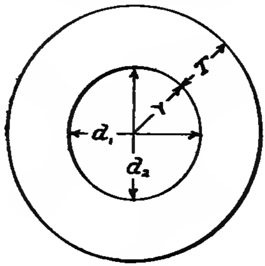


Fig. 40.

To prove the foregoing, assume $d_1 - 2r = 0$, which is the extreme value for r. Here we have the case of the circular cross-section again. Applying formula (150) to Fig. 40 and assigning the values for d_1, d_2, r, and T as follows: Let $d_1 = 1$, $d_2 = 1$, $r = .5$, and $T = .5$. Then $M = 1.274 - .274 + .5 = 1.5$.

Now by formula (52), $M = T + d = 1.5$ also.

When $d_1 = d_2$, i.e., when the cross-section is square, formula (141) becomes $M = 1.274\,d_1 + T$, and formula

(150) becomes $M = 1.274d_1 + T - .574r$. The latter formula, (150), is a general formula, applicable to both square and round cross-sections, formula (141) being correct when $r = 0$, and formula (52), ($M = T + d$), being correct when $2r = d_1 = d_2$.

Formula (150) may be considered as the modulus for converting a square or rectangular winding into a round winding with the same number of turns and resistance, but with different values for d, d_1, d_2, etc., M and T remaining constant.

50. Windings with Elliptical Cross-Sections.

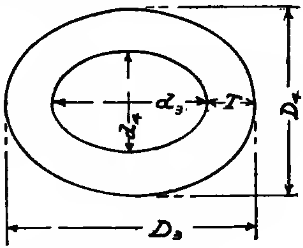


Fig. 41.

Since the area here is simply

$$A = \pi\left(\frac{D_3D_4 - d_3d_4}{4}\right), \tag{151}$$

$$MT = \frac{A}{\pi}\ (133) = \frac{D_3D_4 - d_3d_4}{4}; \tag{152}$$

then

$$T = \frac{D_3D_4 - d_3d_4}{4\,M} \tag{153}$$

$$= \frac{D_3 - d_3}{2} \tag{154}$$

$$= \frac{D_4 - d_4}{2}. \tag{155}$$

$$M = \frac{D_3 D_4 - d_3 d_4}{4\,T} \tag{156}$$

$$= \frac{D_3 D_4 - d_3 d_4}{2\,(D_3 - d_3)}; \tag{157}$$

also
$$M = \frac{D_3 + d_4}{2} \tag{158}$$

$$= \frac{D_4 + d_3}{2}. \tag{159}$$

Since
$$M = \frac{2\,T + d_3 + d_4}{2} \text{ in either case.} \tag{160}$$

From (67),
$$\rho = RMLT.$$

$$\therefore\ \rho = \frac{RL\,(D_3 D_4 - d_3 d_4)}{4}. \tag{161}$$

$$R = \frac{4\,\rho}{L\,(D_3 D_4 - d_3 d_4)}. \tag{162}$$

$$L = \frac{4\,\rho}{R\,(D_3 D_4 - d_3 d_4)}. \tag{163}$$

Radiating surface

$$Sr = \pi L \sqrt{\frac{D_3^2 + D_4^2}{2}}. \tag{164}$$

Substituting values of M from (156) in (80),

$$IN = \frac{4\,ET}{K\,(D_3 D_4 - d_3 d_4)}. \tag{165}$$

51. Windings Whose Cross-Sections have Parallel Sides and Rounded Ends.

From Fig. 42 it is evident that the cross-section of this winding may be resolved into four parts, consisting of two rectangles and two semicircular areas, as in Fig. 43.

The sum of the areas is as follows:

$$A = \pi M T_1 + 2\,T(H - d_5), \tag{166}$$

and $$MT = \frac{A}{\pi},\qquad (133)$$

$$\therefore MT = T[M_1 + .637\,(H - d_5)]. \qquad (167)$$

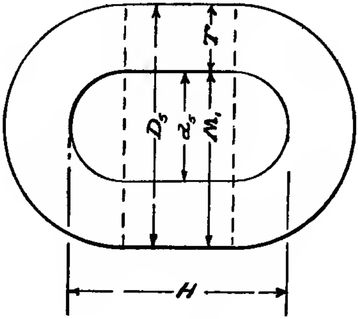


Fig. 42.

Dividing by T,

$$M = M_1 + .637\,(H - d_5); \qquad (168)$$

now $$M_1 = \frac{D_5 + d_5}{2}, \qquad (169)$$

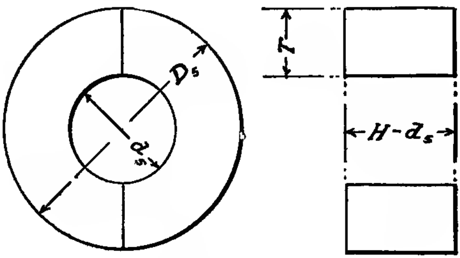


Fig. 43.

$$\therefore M = \frac{D_5 + d_5}{2} + .637\,(H - d_5). \qquad (170)$$

From (170),

$$D_5 = 2\,M + .274\,d_5 - 1.274\,H. \tag{171}$$

$$d_5 = \frac{D_5 - 2\,M + 1.274\,H}{.274}. \tag{172}$$

$$H = \frac{2\,M + .274\,d_5 - D_5}{1.274}. \tag{173}$$

$$T = \frac{D_5 - d_5}{2}. \tag{174}$$

Since
$$MT = T\left[\left(\frac{D_5 + d_5}{2}\right) + .637\,(H - d_5)\right] \tag{175}$$

$$= \frac{\rho}{RL}. \tag{176}$$

$$R = \frac{\rho}{TL\left[\left(\frac{D_5 + d_5}{2}\right) + .637\,(H - d_5)\right]}. \tag{177}$$

$$\rho = RLT\left[\left(\frac{D_5 + d_5}{2}\right) + .637\,(H - d_5)\right]. \tag{178}$$

$$L = \frac{\rho}{RT\left[\left(\frac{D_5 + d_5}{2}\right) + .637\,(H - d_5)\right]}. \tag{179}$$

$$T = \frac{\rho}{RL\left[\left(\frac{D_5 + d_5}{2}\right) + .637\,(H - d_5)\right]}. \tag{180}$$

$$D_5 = .274\,d_5 - 1.274\,H + \frac{2\,\rho}{RLT}. \tag{181}$$

$$d_5 = \frac{D_5 + 1.274\,H - \left(\frac{2\,\rho}{RLT}\right)}{.274}. \tag{182}$$

$$H = \frac{\left(\frac{2\,\rho}{RLT}\right) + .274\,d_5 - D_5}{1.274}. \tag{183}$$

Radiating surface

$$Sr = L\,[2\,(H - d_5) + \pi D_5]. \tag{184}$$

Substituting value of M from (170) in (80),

$$IN = \frac{E}{K\left[\left(\frac{D_5 + d_5}{2}\right) + .637\,(H - d_5)\right]}. \qquad (185)$$

Problems.

33. How many feet of wire may be contained in a winding volume of 1.5 cubic inches if the cross-sectional factor is .000081? 1,852 feet.

34. 367 feet of wire will just fill a bobbin whose available winding volume is .42 cubic inches. What is the cross-sectional factor of the wire? $g^2 = .0001143$.

35. What must be the winding volume of a bobbin to contain 1,000 feet of No. 30 S.C.C. wire whose cross-sectional factor is .000197? $V = .197$.

36. How many ohms per foot in a copper wire 123 circular mils in cross-section? $\omega' = .0842$.

37. What is the mean or average diameter of a winding whose $D = 2$, and $d = 1$? $M = 1.5$.

38. What would be the thickness of the winding in Problem 37? $T = .5$.

39. In Problem 37, what would be the volume of the winding if the length of the winding $L = 3$? $V = 7.069$.

40. When $D = 2$, $T = .6$, (*a*) what is the value of M? (*b*) What is the value of d? (*a*) $M = 1.4$, (*b*) $d = .8$.

41. What resistance would be obtained in a winding space of 3.1416 cubic inches with a wire .0142″ diameter insulated with cotton which increases its total diameter .004″? $\rho = 40.5$.

42. When $D = 3$ and $M = 2.5$, (*a*) what is the value of T? (*b*) Of d? (*a*) $T = .5$, (*b*) $d = 2$.

43. When $T = .5$ and $d = .43$, (*a*) what is the value of D? (*b*) Of M? (*a*) $D = 1.43$, (*b*) $M = .93$.

44. When $M = 2.5$ and $T = .5$, (*a*) what is the value of D? (*b*) Of d? (*a*) $D = 3$, (*b*) $d = 2$.

45. How many ohms per inch in a copper wire .024″ diameter? $\omega'' = .001498$.

46. What is the resistance of a winding where the true outside diameter of the winding is 2 inches, the outside diameter of the insulating sleeve over the core is .55″, and the length of the winding is $2\frac{3}{8}$″, the wire .0065″ diameter insulated with silk which increases its diameter .002″?

$\rho = 1{,}950$.

47. What is the value of K in a copper wire .002″ diameter? $K = .6777$.

48. If the above wire was insulated with 1.5 mil increase of silk, what would be the value of R?

$R = 55{,}322$.

49. What wire should be used to obtain 250 ohms in a bobbin where $MLT = .137$, using single (2-mil) silk insulation? No. 36 wire.

50. What would be the true outside diameter of the above winding if $d = .43$ and $L = 2$? $D = .641$.

51. In Problem 50, how many turns of wire would there be? $N = 4{,}306$.

52. What would be the resistance of the average turn in Problem 50? .0581 ohm.

53. What is the resistance of a winding containing 2,000 turns of No. 24 wire, the average diameter $M = .9$?

$\rho = 12.08$.

54. In the above, what would be the outside diameter if $d = .49$? $D = 1.31$.

55. What would be the ampere-turns in a winding of $2\frac{1}{2}''$ average diameter if wound with No. 22 wire and placed in a 110-volt circuit? $IN = 10{,}427$.

56. In the above, what would be the internal diameter if $D = 4$? $d = 1$.

57. What should be the diameter of a copper wire to produce 3,000 ampere-turns with 220 volts if the average diameter of the winding is $4''$? $\Delta = .01216$.

58. What is the thickness of a winding $3''$ long wound with 3,000 turns of No. 24 S.C.C. wire? $T = .58$.

59. In Problem 58, if $M = 1.7$, (*a*) what is the outside diameter of the winding? (*b*) What is the inner diameter of the winding? (*a*) $D = 2.28$, (*b*) $d = 1.12$.

60. What will be the number of turns in a bobbin where $d = .43$, $L = 2$, if wound to 500 ohms with No. 35 wire, (*a*) with $.002''$ silk? (*b*) With $.004''$ cotton?

(*a*) $N = 8{,}570$, (*b*) $N = 7{,}490$.

61. A winding where $d = .43$, $L = 2$ contains 500 ohms of No. 36 D.S.C. wire with 4-mil insulation. What per cent more turns could be obtained with the same size of wire and the same resistance but by using 2-mil silk insulation? 13.6%.

62. How many layers in a winding where $T = .5$ and $g_v = .00833$? $n = 6$.

63. How many turns per inch where $g_l = .00958$?

$m = 104.4$.

64. In the above two problems, how many turns would be contained in the bobbin if $L = 2$? $N = 1{,}253$.

65. What resistance would be obtained with No. 39 30% G.S.S.S.C. wire in a bobbin where $D = 1.43$, $d = .68$, and $L = 2.5$? $\rho = 238{,}740$

66. What is the intermediate diameter of a winding consisting of No. 40 S.S.C. copper wire and No. 39 30% G.S.S.S.C. wire, in a bobbin where $d = .43$, $D = \frac{5}{8}$, $L = 1$, it being necessary to have a resistance of 4,000 ohms and still have the maximum number of turns of wire? $X = .593.$

67. A winding consisting of three parallel wires connected in multiple has a resistance of 20 ohms. What would be the resistance of the winding if the wires were connected in series? $\rho = 180.$

68. What size of S.C.C. wire must be used in a bobbin where $d = .36$, $D = .648$, and $L = 1\frac{3}{4}$, in order to have two parallel windings of 19 ohms each?

No. 32 S.C.C.

69. What weight of No. 24 S.C.C. wire would be required in a winding consisting of four parallel wires whose joint resistance is 40 ohms? 32.57 lbs.

70. What would be the diameter of a No. $24\frac{1}{2}$ wire in the American wire gauge? $\Delta = .01897.$

71. What is the gauge number of a wire .001″ in diameter? No. 50 (49.88).

72. What will be the permissible insulation on a wire .0074″ diameter in order to wind to a resistance of 100 ohms in a bobbin where $d = .55$, $D = \frac{31}{32}$, and $L = 1\frac{1}{4}$?

$i = .00252.$

73. What will be the resistance of $2\frac{1}{2}$ pounds of S.S.C. wire .0081″ diameter insulated to a diameter of .0083″?

$\rho = 1{,}878.$

74. What will be the total weight of 200 ohms of copper wire .0072″ diameter insulated with 4-mil cotton?

.192 lb.

75. What is the ohms per pound of No. 30 30% G.S. wire insulated with $2\frac{1}{2}$-mil silk? $\theta = 9{,}350$.

76. How many pounds of silk will be required to insulate 100 pounds of No. $27\frac{1}{2}$ copper wire to a 3-mil increase? 5.78 lbs. of silk.

77. How many pounds of No. 24 18% G.S. wire will 25 pounds of cotton insulate to a 5-mil increase?

264.2 lbs.

78. (*a*) What is the mean diameter of a rectangular winding where $D_1 = 4$, $D_2 = 5$, $d_1 = 2$, and $d_2 = 3$? (*b*) What is the value of T? (*a*) $M = 4.185$, (*b*) $T = 1$.

79. In the above, how many ohms of No. 24 S.C.C. wire would the winding contain when $L = 1.5$?

$\rho = 72.57$.

80. What must be the length of a winding where $D_1 = 5$, $D_2 = 5.5$, $d_1 = 3$, and $d_2 = 3.5$ in order to obtain a resistance of 100 ohms with No. 26 S.C.C. wire? $L = .724$.

81. In the above, (*a*) what would be the ampere-turns at 110 volts? (*b*) What would be the number of turns?

(*a*) $IN = 2{,}010$, (*b*) $N = 1{,}824$.

82. In Problem 80, what would be the radiating surface? $Sr = 13.95$.

83. In a bobbin where $d_1 = 2.5$, $d_2 = 4$, radius at corners of core $\frac{1}{2}''$, what will be the value of D_1 and D_2 in order to obtain 1,000 ampere-turns with No. 20 wire at 12 volts? $D_1 = 3.822$, $D_2 = 5.322$.

84. A round winding where $d = 2$, $D = 5$ is to be rewound on to a square core, and the winding on the square core is to contain the same number of turns and resistance as the round winding. What is the value of d_1 and D_1, L being constant in both cases? $d_1 = 1.57$, $D_1 = 4.57$.

85. (*a*) How many turns of No. 27 S.C.C. wire will be contained in a elliptical winding where $D_3 = 1.5$, $D_4 = 2.5$, $d_3 = 1$, $d_4 = 2$, and $L = 3$? (*b*) What will be the resistance of the winding?

(*a*) $N = 2{,}259$, (*b*) $\rho = 53.156$.

86. In Problem 85, (*a*) what will be the radiating surface? (*b*) What will be the ampere-turns at 50 volts?

(*a*) $Sr = 19.43$, (*b*) $IN = 2{,}124$.

87. In a winding where $D_5 = 2$, $d_5 = .75$, $H = 3$, and $L = 2.5$, what size wire with 4-mil insulation would be required to obtain 2,000 turns of wire? $\Delta = .02395$.

88. In the above, what would be the radiating surface?

$Sr = 26.958$.

89. In Problem 87, what would be the ampere-turns at 110 volts with No. 30 wire? $IN = 1{,}453$.

CHAPTER III.

HEATING OF MAGNET COILS.

52. Effect of Heating.

When a current of electricity flows through the winding of an electromagnet, heat is produced due to the current acting against the resistance of the winding, and may properly be called the heat produced by electrical friction.

The amount of heat produced is proportional to the resistance of the winding and the square of the current flowing through it, or, in other words, to the watts lost in the winding.

The heat from the outside layer is radiated rapidly, but the heat from the inner layers has to pass through to the outer layer, core, or washers before it can be dissipated, thus heating the entire winding.

The coil, then, as a whole, radiates the heat gradually at first, but faster and faster as the heat is conducted through the outside layer, until finally the heat is radiated as fast as generated, and equilibrium established. Therefore, it requires considerably more time for a "thick" winding to reach this point than it does for a thinner one, and a thick winding will therefore get hotter inside than a thin one for the same reasons, when under similar electrical conditions.

From this it is evident that the heating of a winding and the time required to reach its maximum is propor-

tional to the thickness of the winding, the square of the current, and the resistance, and is inversely proportional to the radiating surface.

In practice the radiation from the ends of the wind ing and core is not considered, but the total radiation assumed to be from the top or outer layer of the winding. In the average winding about 65% of the heat is radiated from the outer layer, so it is safe to add the other 35%, and thus shorten the calculation.

In practice this method has been found to give as satisfactory results as any, and hence is commonly used.

The heat generated in the coil has the property of increasing the electrical resistance of the winding in a certain ratio for each degree of rise in temperature. This ratio is called the *Temperature Coefficient*, and varies for different metals.

Of course any change of temperature will vary the resistance of the coil, whether due to internal or external influences.

The temperature coefficient for copper wire is .0022, i.e., the resistance of a coil of copper wire will vary .22% for each degree F. of change in temperature. Therefore,

$$\rho_1 = (1 + .0022\,t^\circ)\rho, \tag{186}$$

where t° = rise in temperature in degrees F.

EXAMPLE. — A coil of copper wire has a resistance of 100 ohms at 75° F. (*a*) What will be its resistance at 100° F.? (*b*) At 32° F.?

SOLUTION. — (*a*) $100^\circ - 75^\circ = 25^\circ$ rise $= t^\circ$.

$(1+(.0022 \times 25)) \times 100 = 1.055 \times 100 = 105.5$ ohms. *Ans.*

(*b*) $75° - 32° = 43°$ drop.

$$\rho = \frac{\rho_1}{1 + .0022\,t°}. \qquad (187)$$

$$\therefore \rho = \frac{100}{1 + (.0022 \times 43)}$$

$$= \frac{100}{1.0946} = 91.36 \text{ ohms.} \quad \textit{Ans.}$$

The radiation of heat from a winding depends upon so many things that in practice it is assumed that the average rise in temperature in the winding will be 100° F. when the winding is radiating a certain number of watts per square inch continuously, the rate of radiation depending on the thickness of the winding.

Thus, when an ordinary telephone ringer magnet is radiating approximately .9 watts per square inch continuously, the rise in temperature will be approximately 100° F., while a winding $4\frac{1}{8}''$ in diameter, $7''$ long, and $1\frac{7}{16}''$ thick will rise in temperature 100° F. when the winding is radiating .33 watts per square inch continuously.

The rise in temperature in a winding is directly proportional to the rate of continuous radiation. Thus, a winding that will rise in temperature 100° F. when radiating .5 watts continuously will rise 200° F. when radiating 1 watt per square inch.

The permissible rise in temperature depends entirely on the temperature of the place where the winding is to be used. In any case, the temperature of the surrounding air must be deducted from the limiting temperature.

When several coils of the same dimensions, but for use with different voltages, are to be made, it is best

to test one coil and ascertain the rise in temperature and the watts per square inch for different periods of time. From the data thus obtained the proper wire may be easily calculated for the other windings at different voltages, to obtain the maximum ampere-turns without overheating.

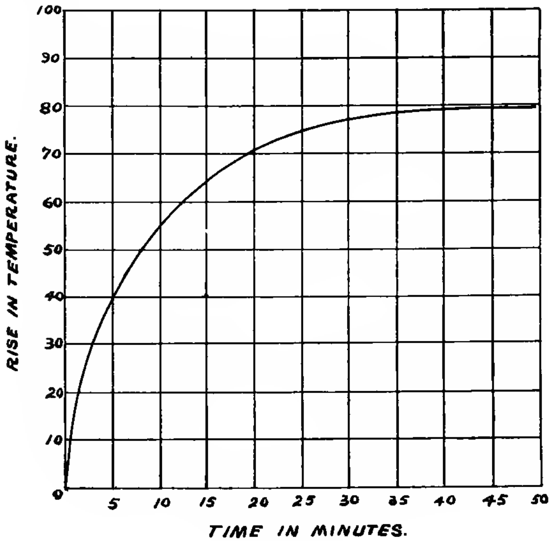


Fig. 44.

Fig. 44 shows the curve obtained from such a test. It will be seen that the coil heats very rapidly at first and then gradually reaches equilibrium when the heat is radiated as fast as generated. It will also be noted that the temperature will be known at the end of any time.

To make the test, use a mil-ammeter and voltmeter. The source of current must be of constant voltage to give good results.

Fig. 45 shows the connections for the test.

By Ohm's law (3)

$$\rho = \frac{E}{I},$$

and from formula (186) transposed, the rise in temperature

$$t^\circ = \frac{\rho_1 - \rho}{.0022\,\rho}. \tag{188}$$

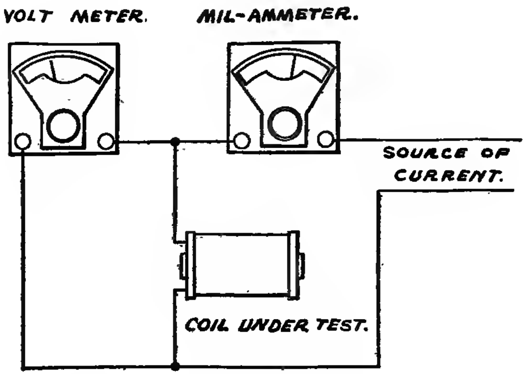


Fig. 45.

This may be noted at the end of every few minutes, and a curve plotted through the points thus found.

The watts are equal to EI (see formula 6), and the watts per square inch

$$W_s = \frac{EI}{Sr}, \tag{189}$$

which for a round magnet is

$$W_s = \frac{EI}{\pi DL}. \tag{190}$$

The resistance of a winding to be used on any voltage is then

$$\rho_1 = \frac{E^2}{W_s Sr}, \tag{191}$$

or

$$\rho_1 = \frac{E^2}{W_s \pi DL}, \tag{192}$$

for a round winding.

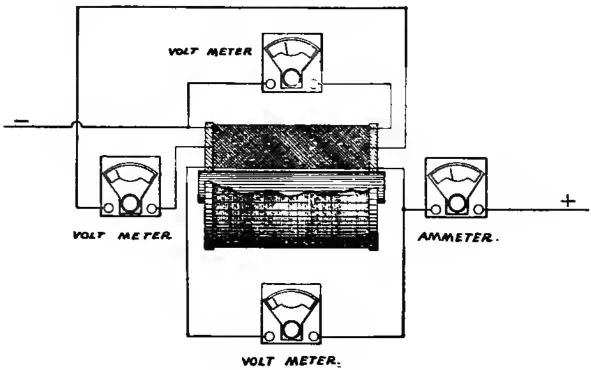


Fig. 46.

If the current is to be kept constant in the winding the voltage will vary, but the rise in temperature for any period of time may be found by means of the test as explained above.

The ratio of heating between the inside, middle, and outside layers may be determined by the above method also, by connecting wires to both ends of each layer to be tested, as in Fig. 46.

53. Relation between Magnetomotive Force and Heating.

Where constant voltage is used, the ampere-turns vary directly as the watts, since voltage and turns are constant, and

$$\text{Ampere-turns} = \text{amperes} \times \text{turns},$$
$$\text{watts} = \text{amperes} \times \text{volts},$$

therefore, the ratio between magnetomotive force and heating due to the current in the winding is constant, since the current varies directly with the resistance.

Hence the watts and ampere-turns will fall off in the same ratio until the heat is radiated as fast as generated, when they will become constant. For this reason magnets which are to work continuously should be calculated to do the work at the limiting temperature.

First determine, from the dimensions of the winding, how many watts the coil will radiate from its surface for the required rise in temperature. Then the resistance at the limiting temperature will be

$$\rho_1 = \frac{E^2}{W}, \tag{193}$$

and the resistance at the air temperature

$$\rho = \frac{\rho_1}{1 + .0022\, t^\circ} = \frac{\rho_1}{1.22} \tag{194}$$

for 100° rise.

Thus the cold resistance for 150° rise would be,

$$\frac{\rho_1}{1 + .0022\, t^\circ} = \frac{\rho_1}{1.33} = \rho.$$

With constant current, ampere-turns and voltage and watts vary directly with the resistance. Since the resistance increases as the heat increases, thus increasing the generation of heat, it is also very important that the ampere-turns should be calculated at the limiting temperature.

From permissible watts per square inch at limiting temperature calculate the resistance at limiting temperature, then the cold resistance

$$\rho = \frac{\rho_1}{1 + .0022\, t^\circ}. \qquad (194)$$

A winding of fixed dimensions will contain the same number of ampere-turns at a given rise in temperature, no matter what the resistance or voltage may be.

The above is of course on the assumption that the relation between diameter of wire and thickness of insulation is constant.

In practice the ratio between diameter and thickness of insulation is not exactly constant, so the above rule is only approximate.

Therefore, if a certain coil will contain a certain number of ampere-turns with a certain wire at a certain voltage, approximately the same number of ampere-turns will be obtained with any other wire, if the voltage varies in the same ratio as the square of the diameter of the wire.

From (82) $$\Delta = \sqrt{\frac{cINM}{E}},$$

which is the diameter of copper wire for the given number of ampere-turns.

This, however, does not take into consideration the heating effect of the coil. In order to control the heating, the resistance must have a certain value, and the resistance changes with the thickness of the insulation on the wire. Therefore, the volume of the bobbin, the radiating surface, and the thickness of the insulation must be known.

EXAMPLE. — Given bobbin, find the exact diameter of wire to use with given insulation that will give the greatest number of ampere-turns without overheating the coil.

Assume that a coil will radiate 1 watt per square inch for a rise of 100° F., and the dimensions are as follows:

Core $\frac{3}{8}''$, $d = .43$, $D = 1$, $L = 2$, and the voltage $= 50$; then the area is $\pi DL = 6.2832$ square inches.

The resistance of the winding at the limiting temperature will be

$$\rho_1 = \frac{E^2}{W_s \pi DL}, \tag{195}$$

where $W_s =$ watts per square inch.

$$\therefore \quad \rho_1 = \frac{50^2}{6.2832} = \frac{2,500}{6.2832} = 397.9 \text{ ohms,}$$

and $$\rho = \frac{397.9}{1.22} = 326.5 \text{ ohms}$$

at the temperature of the surrounding air.

Now $$MLT = \left(\frac{D^2 - d^2}{4}\right) L \tag{196}$$

$$= \left(\frac{1 - .185}{4}\right) \times 2 = .407.$$

From (68), $R = \frac{\rho}{MLT} = \frac{326.5}{.407} = 8,022.$

Assume that $\qquad i = .002.$

Since $\quad R = \dfrac{K}{g^2}$ (65), and $K = \dfrac{c}{\Delta^2}$ (63),

$$R = \frac{c}{\Delta^2 g^2}. \tag{197}$$

(See (72).)

$$\therefore\ \Delta^2 g^2 = \frac{c}{R}, \qquad \text{and } \Delta g = \sqrt{\frac{c}{R}}.$$

Since $\quad g = \Delta + i$ (46), $\qquad \Delta^2 + \Delta i = \sqrt{\dfrac{c}{R}}$;

completing the square,

$$\Delta^2 + \Delta i + \frac{i^2}{4} = \sqrt{\frac{c}{R}} + \frac{i^2}{4},$$

$$\Delta + \frac{i}{2} = \left[\sqrt{\frac{c}{R}} + \frac{i^2}{4}\right]^{\frac{1}{2}},$$

and

$$\Delta = \left[\sqrt{\frac{c}{R}} + \frac{i^2}{4}\right]^{\frac{1}{2}} - \frac{i}{2}. \tag{198}$$

$$\therefore\ \Delta = \left[\sqrt{\frac{.00000271}{802.2}} + \frac{.00004}{4}\right]^{\frac{1}{2}} - \frac{.002}{2}$$

$$= \sqrt{.00005819 + .00001} - \frac{.002}{2}.$$

$$\therefore\ \Delta = .00826 - .001 = .00726. \quad \textit{Ans.}$$

To shorten the calculation, substitute $\dfrac{\rho}{MLT}$ for R, then

$$\Delta = \left[\sqrt{\frac{cMLT}{\rho}} + \frac{i^2}{4}\right]^{\frac{1}{2}} - \frac{i}{2}, \tag{199}$$

or

$$\Delta = \left[\sqrt{\frac{cL(D^2 - d^2)}{4\rho}} + \frac{i^2}{4}\right]^{\frac{1}{2}} - \frac{i}{2}. \tag{200}$$

Now, since $\rho_1 = \dfrac{E^2}{W_s \pi DL}$, (192)

$$\rho = \frac{E^2}{W_s \pi DL(1 + .0022\, t^\circ)} \tag{201}$$

for any rise in temperature.

The complete formula may be written

$$\Delta = \left[\sqrt{\frac{(1+.0022\,t^{\circ})\,c\,W_s\,\pi D L^2 (D^2 - d^2)}{4\,E^2}} + \frac{i^2}{4}\right]^{\frac{1}{2}} - \frac{i}{2}, \quad (202)$$

which gives the exact diameter of bare copper wire, which will give the maximum number of ampere-turns within the limiting heating conditions.

Now, to prove the last formula, take the same example as above; then

$$\Delta = \left[\sqrt{\frac{1.22 \times .00000271 \times 1 \times 3.1416 \times 1 \times 4 \times .815}{4 \times 2{,}500}} + .00001\right]^{\frac{1}{2}} - .001$$

$$= \left[\sqrt{\frac{.00003386}{10{,}000}} + .00001\right]^{\frac{1}{2}} - .001$$

$$= \sqrt{.00005819 + .00001} - .001.$$

$$\therefore \Delta = .00826 - .001 = .00726, \textit{ Ans.},$$

which is the same result as obtained before.

Substituting value of MT from (134) in (199),

$$\Delta = \left[\sqrt{\frac{cL\left[(D_1 D_2 - d_1 d_2) - .8584\,T^2\right]}{\pi\rho}} + \frac{i^2}{4}\right]^{\frac{1}{2}} - \frac{i}{2} \quad (203)$$

for square or rectangular windings.

Substituting value of MT from (152) in (199),

$$\Delta = \left[\sqrt{\frac{cL\,(D_3 D_4 - d_3 d_4)}{4\,\rho}} + \frac{i^2}{4}\right]^{\frac{1}{2}} - \frac{i}{2} \quad (204)$$

for elliptical windings.

Substituting value of MT from (175) in (199),

$$\Delta = \left[\sqrt{\frac{cLT\left[\left(\frac{D_5 + d_5}{2}\right) + .637\,(H - d_5)\right]}{\rho}} + \frac{i^2}{4}\right]^{\frac{1}{2}} - \frac{i}{2} \quad (205)$$

for windings with parallel sides and rounded ends.

To find the number of ampere-turns, use formula (80),

$$IN = \frac{E}{KM},$$

or

$$IN = \frac{E\Delta^2}{cM}. \qquad (206)$$

Therefore, the maximum number of ampere-turns that may be obtained continuously, in any winding volume with any voltage, with given insulation and at the limiting temperature, is

$$IN = \frac{E\left(\left[\sqrt{\frac{(1 + .0022\, t^\circ)\, c W_s\, Sr\, MLT}{E^2}} + \frac{i^2}{4}\right]^{\frac{1}{2}} - \frac{i}{2}\right)^2}{cM}. \qquad (207)$$

Here Sr = radiating surface, and is equal to πDL for round windings,

$$2\, L\, [(d_1 + d_2) + 1.5708\, (D_1 - d_1)] \quad (147)$$

for square or rectangular windings,

$$\pi L \sqrt{\frac{D_3^2 + D_4^2}{2}} \quad (164)$$

for elliptical windings, and

$$L\, [2\, (H - d_5) + \pi D_5] \quad (184)$$

for windings with parallel sides and rounded ends.

The general relations between watts, voltage, resistance, current, and radiating surface are expressed by the following formulæ, the values being taken at the normal temperature, or the temperature of the surrounding atmosphere.

Let W_s = watts per square inch,
W = total watts ;

then $$W_s = \frac{W}{Sr} \cdot \tag{208}$$

Since $$W = I^2\rho \ (4) \ = \frac{E^2}{\rho} (5) = EI \ (6),$$

$$W_s = \frac{I^2\rho}{Sr} \tag{209}$$

$$= \frac{E^2}{Sr\rho} \tag{210}$$

$$= \frac{EI}{Sr}, \tag{211}$$

$$E = \sqrt{W_s\rho Sr} \tag{212}$$

$$= \frac{W_s Sr}{I}, \tag{213}$$

$$Sr = \frac{I^2\rho}{W_s} \tag{214}$$

$$= \frac{E^2}{W_s\rho} \tag{215}$$

$$= \frac{EI}{W_s}, \tag{216}$$

$$\rho = \frac{W_s Sr}{I^2} \tag{217}$$

$$= \frac{E^2}{W_s Sr} \cdot \tag{218}$$

$$I = \sqrt{\frac{W_s Sr}{\rho}} \tag{219}$$

$$= \frac{W_s Sr}{E} \cdot \tag{220}$$

To find the above values at the limiting temperature, multiply the watts per square inch W_s, by $(1 + .0022\, t^\circ)$.

Thus $$E = \sqrt{W_s\rho Sr\,(1 + .0022\, t^\circ)}. \tag{221}$$

In some forms of apparatus an electromagnet is employed to raise a weight and then sustain it indefinitely.

Since the magnet requires less current to sustain the weight than to attract it from a distance, the winding is designed to carry the sustaining current continuously, the heating due to the attracting current not being considered unless the attracting current is very much greater than the sustaining current, as the former is on only momentarily.

This principle is employed in self-starting motor rheostats.

If E represents the attracting voltage, and E_1 the sustaining voltage,

$$W_s = \frac{E_1^2 ING^2}{SrTLE\,(1 + .0022\,t^\circ)}, \tag{222}$$

$$IN = \frac{W_s SrTLE\,(1 + .0022\,t^\circ)}{E_1^2 g^2}. \tag{223}$$

If the attracting voltage and the sustaining voltage are equal,

$$W_s = \frac{EINg^2}{SrTL(1 + .0022\,t^\circ)}, \tag{224}$$

$$IN = \frac{W_s SrTL\,(1 + .0022\,t^\circ)}{Eg^2}. \tag{225}$$

54. Advantage of Thin Insulating Material.

If a bobbin was wound with bare wire with no insulation, the ampere-turns would reach a maximum when the watts were at their maximum. In this, of course, the ampere-turns are meant to be the number of turns in the coil multiplied by the current which would pass through the bare wire when suspended in air.

In practice the wire is insulated, which increases the total volume considerably, and therefore, in order to obtain

the same resistance with the insulated wire, in a given bobbin, as would be obtained with bare wire, a shorter length of insulated wire with a smaller cross-section of copper must be used. When this is done, however, the resistance of the average turn is increased, and therefore the ampere-turns will be reduced and will not be at their maximum when the watts are maximum, as with the bare wire.

The ampere-turns will therefore reach a maximum when the voltage divided by the resistance of the average turn produces a maximum.

It is therefore obvious that the efficiency of a winding increases as the thickness of the insulation decreases.

For this reason, silk-covered wire is much more efficient than cotton-covered wire, although its cost is greater.

Whatever is saved in first cost of winding with any fixed insulation, is paid for in the cost of operating and at exactly the same rate as the saving in first cost if constant voltage is used.

55. Work at End of Circuit.

When an electromagnet is to be connected at the end of a line of considerable resistance, the winding of the magnet should have slightly less resistance than the line, in order to do the most work, providing, of course, that the winding volume is great enough to prevent the winding from becoming overheated.

The reason for this is, that if the line has the greater resistance, it will absorb more voltage than the coil, with the same current, thus absorbing more watts.

Again, if the coil contains more resistance than the line,

the resistance of the average turn will have been so increased by the use of finer wire that the ampere-turns will be greatly decreased, the dimensions of the winding being the same in both cases.

The voltage across the terminals of the electromagnet winding is

$$E = \frac{E_1 \rho}{\rho + \rho_1}, \tag{226}$$

where E = voltage across coil,
E_1 = voltage of line,
ρ = resistance of coil,
ρ_1 = resistance of line.

As an example, assume that an electromagnet is to operate at the end of a 220-volt circuit, the resistance of the line being 250 ohms, and the dimensions of the magnet winding as follows: $MLT = .2$. $M = 1$. Watts per square inch permissible.

This is shown calculated for both bare and single silk-insulated wire. The difference in ampere-turns between the bare and insulated wire will be noted, also the fact that the ampere-turns are at a maximum with the watts for bare wire, but not for insulated wire.

Bare Wire.

Wire No.	K	R	ρ	E	IN	W
32	.0429	679	135.8	77.5	1,805	44.2
33	.0541	1,080	216	102	1,880	48.1
34	.0682	1,715	343	127	1,860	47

Here the maximum falls between No. 33 and No. 34 for both ampere-turns and watts.

Single Silk-Covered Wire.

$$i = .0025.$$

Wire No.	K	R	ρ	E	IN	W
33	.0541	589	117.8	70.5	1,320	42
34	.0682	880	176	91	1,330	47
35	.086	1,307	261.4	112	1,305	48

Here the ampere-turns are maximum with No. 34 wire, while the watts are maximum with No. 35 wire.

Therefore, calculate the size of wire to use assuming the resistance of the coil to be equal to the resistance of the line and battery, or source of energy, and then try the next larger size of wire, selecting the wire which gives the greatest number of ampere-turns.

Now,
$$E = \frac{E_1 \rho}{\rho + \rho_1}, \tag{226}$$
and
$$\rho = \frac{cMLT}{\Delta^2 (\Delta + i)^2}. \tag{227}$$
(See 62.)

Substituting value of ρ from (227) in (226),

$$E = \frac{E_1}{\left(\frac{\rho_1 \Delta^2 (\Delta + i)^2}{cMLT}\right) + 1}. \tag{228}$$

The ampere-turns are maximum when $\frac{E\Delta^2}{cM}$ is maximum.

$$\therefore\ IN = \frac{E_1}{\left(\frac{\rho_1 (\Delta + i)^2}{TL}\right) + \frac{cM}{\Delta^2}}. \tag{229}$$

If the magnet is to carry the current continuously, the bobbin must be made large enough to radiate the heat.

A magnet should always be so designed that it will stand the total voltage of the line without overheating.

It is a mistake to place a resistance in series with a magnet, for the power lost in the dead resistance is nearly in direct proportion to the relative resistances of the magnet and the dead-resistance coil, and therefore for the same total energy the magnet will be much weaker than when designed to have the full voltage without overheating. Consequently this practice is wrong. Furthermore, the cost of operating varies as I^2, so it is seen that the higher the resistance, the more economical will be the operating of the magnet.

Problems.

90. The resistance of a winding is 87.5 ohms at 70° F. (*a*) What will be its resistance at 160° F.? (*b*) At −10° F.?
(*a*) $\rho_1 = 104.815$, (*b*) $\rho = 74.4$.

91. What would be the temperature coefficient of a wire which changed from 320 ohms at 130° F. to 310 ohms at 70° F.? .000538.

92. The resistance of a copper wire winding at 80° F. is 25 ohms. (*a*) What will be its resistance at 0° F.? (*b*) At 100° F.? (*a*) $\rho = 21.26$, (*b*) $\rho_1 = 26.1$.

93. What would be the watts per square inch where $E = 110$, $I = .3$, and $Sr = 66$? $W_s = .5$.

94. What would be the permissible resistance in a winding where $D_1 = 4$, $D_2 = 4.5$, $d_1 = 2.5$, $d_2 = 3$, $L = 2$, $E = 500$, and $W_s = .6$? $\rho = 13{,}260$.

95. In the above, (*a*) what would be the proper size of wire to use? (*b*) What would be the ampere-turns?
(*a*) No. 35 S.S.C., (*b*) $IN = 1365$.

96. What must be the resistance of a winding at 68° F., to radiate .5 watt per square inch at 150° F., with 500 volts, assuming the radiating surface to be 93 square inches? $\rho = 4{,}557$.

97. What would be the theoretically exact diameter of wire to use with 4-mil insulation at 68° F., in order that the average temperature of the coil will not rise above 150° F. during continuous service on a 110-volt circuit, assuming radiating surface to be 40 square inches, $W_s = .7$, and $MLT = 3$? $\Delta = .01036$.

98. What would be the maximum ampere-turns at 150° F. in a winding where $d_1 = 2$, $d_2 = 4$, $D_1 = 3.5$, $D_2 = 5.5$, and $L = 4.5$, voltage 110, insulation 4-mil cotton, and the watts per square inch = .5, assuming normal temperature to be 68° F? $IN = 2{,}410$.

99. How many watts per square inch at the limiting temperature (*a*) where $I = .5$, $\rho = 100$, $Sr = 50$? (*b*) Where $E = 110$, $\rho = 220$, $Sr = 60$? (*c*) Where $E = 50$, $I = .5$, $Sr = 40$?

(*a*) $W_s = .5$, (*b*) $W_s = .917$, (*c*) $W_s = .625$.

100. What would be the safe voltage, allowing .5 watt per square inch at limiting temperature, (*a*) where $\rho = 100$, $Sr = 19$? (*b*) Where $I = .2$, $Sr = 15$?

(*a*) $E = 30.82$, (*b*) $E = 37.5$.

101. What would be the safe current at 68° F., allowing .8 watt per square inch at the limiting temperature, (*a*) where $\rho = 200$, $Sr = 25$? (*b*) Where $E = 500$, $Sr = 90$? (*a*) $I = .316$, (*b*) $I = .144$.

102. An electromagnet is designed to attract its armature and load on a 220-volt circuit, and then to maintain the load at 140 volts. The required ampere-turns

are found to be 5,100. The dimensions of the winding are as follows: $d = 1\frac{5}{16}$, $D = 3\frac{1}{4}$, $L = 4\frac{1}{8}$, and the space factor $g^2 = .000323$. What will be the watts per square inch at the limiting temperature? $W_s = .872$.

103. In Problem 102, how many ampere-turns would be obtained at 220 volts if the watts per square inch at 140 volts were increased to .9, and the space factor $g^2 =$ .00041? $IN = 4{,}147$.

104. An electromagnet with dimensions $d = .55$, $D =$ 1.03, $L = 3$, is to be placed in a 24-volt circuit which has a line resistance of 10 ohms. What wire should be used to give the maximum ampere-turns, assuming the insulation to be 4-mil cotton? No. 24.

CHAPTER IV.

ELECTROMAGNETS AND SOLENOIDS.

56. Forms of Electromagnets.

THE best form of magnetic circuit is the ring, Fig. 47, as it has the minimum magnetic leakage; but it is not the best form of electromagnet on account of the space lost in the winding, for if the wire is wound evenly in layers on the inside of the ring, there will be considerable space between the turns on the outside of the ring.

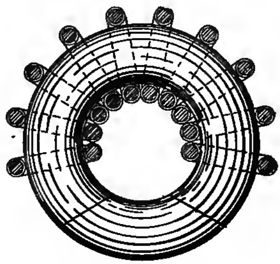


Fig. 47.

Again, if the turns are wound evenly and close together on the outside of the ring, the turns will be crowded or bunched on the inner side, thus increasing the length of the mean turn, and decreasing the ampere-turns.

The *Bar Electromagnet*, Fig. 48, has the same general field as the permanent bar magnet in Fig. 1, but the core is of soft iron and is surrounded by a coil of wire, Fig. 49.

The bar electromagnet is not efficient, however, unless the armature is bent into the form of U so as to complete the magnetic circuit. Even then there will be much leak-

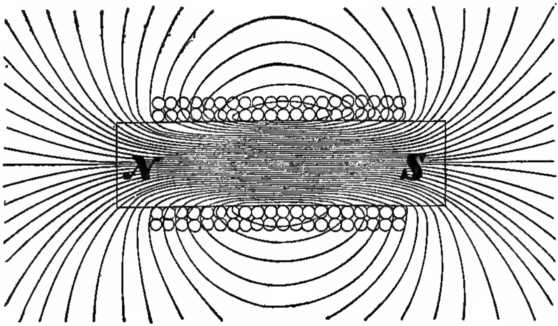


Fig. 48.

age between the parallel legs of the armature, unless they are very short.

If only one pole attracts the armature, i.e., is near and the other pole distant, the magnet will be very weak on

Fig. 49.

account of the high reluctance of the air through which the magnetic flux has to pass.

The *Horseshoe Electromagnet*, Fig. 50, is the most efficient type for a short range of action, but as this form

is rather inconvenient to make and therefore expensive, the practical horseshoe electromagnet is made of three pieces besides the armature.

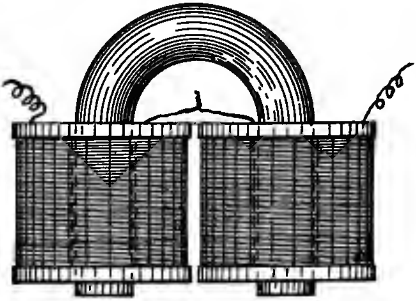


Fig. 50.

In this form, Fig. 51, the wire is wound directly on to the cores, and they are then fastened to the yoke, or "back iron," as it is often called.

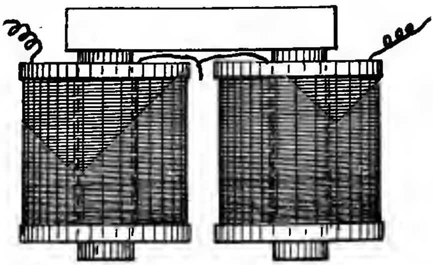


Fig. 51.

While this form is very efficient, there is a loss due to the joints between the cores and yoke.

The *Iron-clad Electromagnet*, Fig. 52, is really a form of horseshoe electromagnet with one of its poles in the form of a shell surrounding the other pole. This type is usually round, and consists of a solid piece of iron or steel with a deep groove turned inside to receive the winding.

The iron-clad electromagnet has the advantage of being mechanically protected, and is also free from external inductive influences, but it has the decided disadvantage of having little or no ventilation, depending

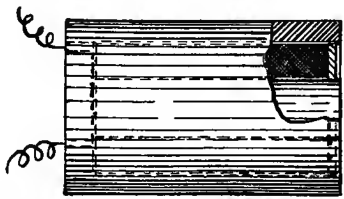


Fig. 52.

upon the heat being conducted through the air inside the shell before it can be radiated by the iron. Of course a great deal of the heat is conducted through the core to the outside, but entangled air is a very poor conductor of heat for the outside of the winding. This may be largely overcome by cutting away a portion of the shell so as to allow a circulation of air next to the winding, but this would increase the reluctance of the magnetic circuit by decreasing the amount of iron.

The magnetic leakage is also great when the armature is a comparatively short distance from the poles.

This type is used extensively in telephone switchboard apparatus where the current is of short duration and the

range of action is also very short. Its principal feature in this case is that it is not affected by external magnetic influences.

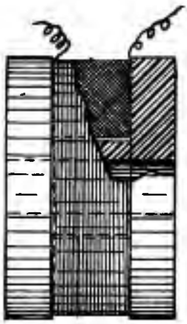


Fig. 53.

Another form of electromagnet called the *Circular Electromagnet*, Fig. 53, is used in magnetic clutches.

This consists simply of a ring of iron or soft steel with a groove cut in its periphery to receive the winding.

When excited, one of its faces is north-seeking and the other south-seeking.

There are many other types, but the ones described are more commonly used.

57. Direction of Flux in Core.

In Fig. 10 was shown the relation of direction of current in a wire to direction of lines of force about a wire.

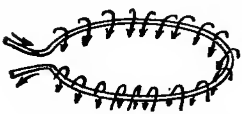


Fig. 54.

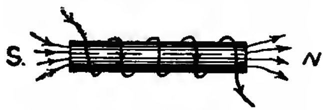


Fig. 55.

In the electromagnet where the wire is coiled about the core, the same relation holds as in Fig. 54 and Fig. 55.

There are a great many rules to aid in remembering this law, but probably the simplest one is the analogue of the corkscrew or ordinary right-hand screw.

As a corkscrew is turned to the right it enters the cork. Simply consider the cork as the north-seeking pole, and the direction of rotation as the direction of the current.

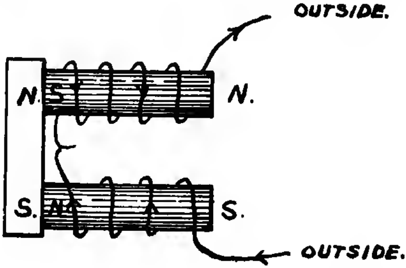


Fig. 56.

In two-pole magnets of the type shown in Fig. 51, it is convenient to make the windings of both bobbins in the same direction. When they are fastened to the yoke and connected, however, the two inside terminals must be connected, as otherwise the two windings would be acting in opposition to one another.

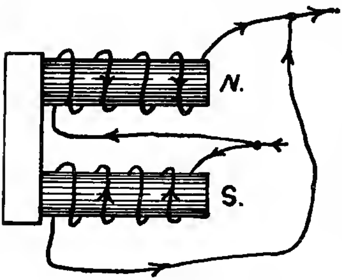


Fig. 57.

Fig. 56 shows this principle.

In electromagnets and solenoids having but one bobbin, the direction of the winding is immaterial.

When the coils are connected in multiple (to reduce the time constant, or for use on less voltage than for which they were designed to work in series), the inside terminal of each winding must be connected to the outside terminal of the other for the same reasons as explained above. Fig. 57 shows the method of connecting the two windings of an electromagnet in multiple.

58. Action of an Electromagnet.

When the armature is separated from the poles of an electromagnet, it is attracted by the poles, and the magnet is called an *Attractive Electromagnet.* If the armature is at a considerable distance from the poles, the reluctance of the air gap between the armature and the poles may equal or exceed the reluctance between the two poles, and if equal, only half of the total magnetic flux will pass through the armature.

As the armature approaches the poles, the reluctance of the air gap becomes less and the attraction becomes stronger, and there is also less leakage between the poles, until finally, when the armature touches the poles, there is comparatively little reluctance between the poles and the armature, and practically no leakage between the poles. The magnet is then said to be a *Portative Electromagnet.*

The reluctance in air between two surfaces is $R = \frac{l}{A}$, since $\mu = 1$ in air. Therefore, as A increases, R decreases, l remaining constant. If the reluctance of the air gap is very great as compared with the reluctance of the rest of the magnetic circuit, we may neglect the latter reluctance, and also, by assuming that the leakage ratio is constant, concentrate our attention on the effect in the air gap alone. Since the flux density is constant for any cross-section of core, with constant M.M.F., increasing the area of the poles of a magnet also increases the pull of the magnet, since the pull is proportional to B^2A.

While not strictly correct, the above is nearly correct for comparatively long air gaps.

The best form of pole for producing strong fields is a conical pole piece of 120° aperture, which will give a field of approximately 250,000 lines per square inch over an extent of several square millimeters. The radius of the pole face should be about $\frac{1}{8}''$, making the pole nearly in the form of a parabola.

59. Calculation of Traction.

When it is desired to construct an electromagnet, the principle data given is the *Traction*, or pull.

The formula for the pull is

$$P = \frac{B^2 A}{72{,}134{,}000} \qquad (230)$$

The table on p. 150 is calculated from this formula upon the assumption that there is a uniform distribution of lines of force over the area considered, and that there is no magnetic leakage.

The practical working densities for different grades of iron and steel, providing the permeability does not fall below 200–300, are approximately as follows:

Wrought iron	90,000
Cast steel	85,000
Mitis iron	80,000
Ordinary cast iron	50,000

Therefore, to find the size of core to use, select from the table the pull in pounds per square inch opposite the working density, and dividing the required pull by the pounds per square inch gives the area of the pole. Likewise, to find the pull when the value of B and the area of

the pole are known, find the pull in pounds per square inch from the table and divide by the area of the core.

EXAMPLE. — A magnet is to be designed that will sustain a weight of 10 pounds, the core material being cast steel. What should be the area of the core at the poles?

SOLUTION. — The practical working density for cast steel is $B = 90{,}000$. In table, when $B = 90{,}000$, $P = 112.3$, therefore the area of the core

$$A = \frac{10}{112.3} = .089 \text{ square inches,}$$

or a core .336″ in diameter.

In order to obtain fair results, the armature and pole must be accurately faced, as a non-uniform distribution of the lines and increased reluctance may diminish the traction.

On the other hand, a diminished area of contact will increase the traction providing the total flux passes through the joint.

A two-pole magnet will sustain twice the weight of a so-called single-pole magnet, for the same intensity of induction, as the magnetic flux is utilized twice and there is twice the pole area.

The larger the cores of an electromagnet the greater will be the strength, providing the cross-sections of the armature and yoke vary in the same ratio as the cross-section of the cores, the outside diameter of the winding, the resistance, and the length of the magnetic circuit remaining constant; for while the ampere-turns will fall off as the core increases in diameter, the cross-section of the core and B^2 increase faster than the ampere-turns decrease.

60. Solenoids.

The solenoid or coil and plunger magnet consists of an electromagnet with the core free to move and which acts as the armature. This is based upon the tendency of the lines of force to take the shortest path; and as the

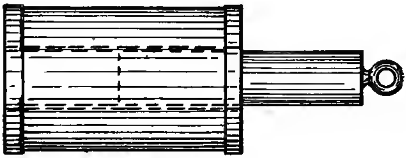


Fig. 58.

iron offers less reluctance than the air, the force greatly increases, and the core is drawn or pulled into the center of the winding. The solenoid is commonly used where a long range of action is required.

The simple solenoid, Fig. 58, is the least efficient form, although it is commonly used. Its efficiency may be rated with that of the bar electromagnet when only one pole is used to attract the armature, as the only return circuit for the lines of force is the surrounding air.

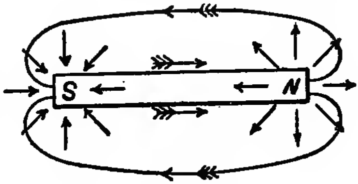


Fig. 59.

The magnetic field of a solenoid is not perfectly uniform, but is nearly so at the center, decreasing towards the ends, where it is weakest on account of the demagnetizing effect of the poles, which react as in Fig. 59.

The feathered arrows represent the direction of the lines of force produced by the solenoid, and the plain arrows the direction of the lines of force due to the reaction of the poles.

When an iron core is placed inside of a solenoid the demagnetizing action is greatly increased, but it decreases as the ratio of length to diameter increases.

61. Action of Solenoids.

In the case of the simple solenoid, the pole induced at the lower end of the plunger as it approaches and enters the solenoid is attracted and drawn farther in, thus decreasing the reluctance of the magnetic circuit and in-

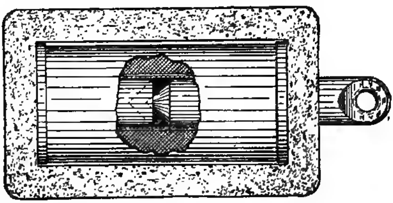


Fig. 60.

creasing the flux and consequently the attraction; the attraction being maximum when the end of the plunger reaches the center of the solenoid.

The most efficient and generally useful form is the *Iron-clad Solenoid*, or "*Plunger Electromagnet*," Fig. 60.

In this form the magnetic return circuit consists of iron of sufficient cross-section to make the reluctance very low.

The frame is usually a wrought-iron forging or steel casting, although cast iron serves very well where the air gap is great, as the reluctance of the air gap is so great that the reluctance of the cast-iron frame is very low by comparison. The spool or bobbin is usually made of brass, the tube of the spool extending clear through the upper end of the iron frame; this keeps the core or plunger from sticking to the iron, while the tube is still too thin to introduce much reluctance into the circuit at that point.

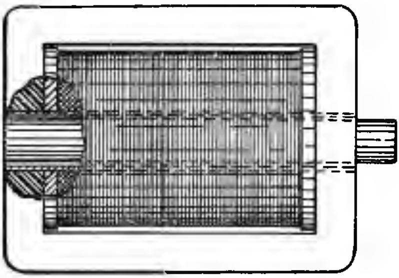


Fig. 61.

In some solenoids of this type the plunger passes clear through the frame at both ends, as in Fig. 61; but the form with the iron stop is the strongest, as there is no extra air gap, and the attraction is between the iron stop and the plunger, instead of between the walls of the hole through the frame and the plunger.

As a portative magnet, the one with the stop will hold many times the weight held by the other form, and as a tractive magnet it is also much stronger, especially if the stop and plunger are V-shaped, as in Fig. 62.*

* W. E. Goldsborough, Electrical World, Vol. XXXVI., July 28, 1900.

Fig.* 63 shows the best condition for the V-shaped gap. In the case of the iron-clad solenoid, the action is a combination of the simple solenoid and electromagnet, the

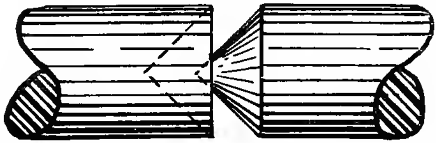


Fig. 62.

attraction reaching a maximum when the plunger completely closes the magnetic circuit.

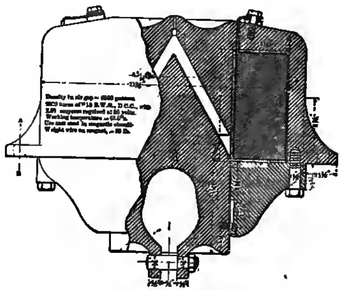


Fig. 63.

62. Polarized Magnets.

This form of electromagnet is the same as the common form with the exception that its armature is a permanent magnet, or else the entire electromagnet is influenced by a permanent magnet.

Fig. 64 and Fig. 65 show two forms where the armatures are permanent magnets made from hardened steel.

* W. E. Goldsborough, Electrical World, Vol. XXXVI., July 28, 1900.

In Fig. 64 the armature is pivoted at one end, and in Fig. 65 the armature is pivoted in the center. One great advantage of this form of electromagnet is that the

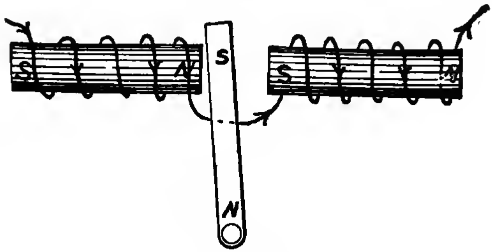


Fig. 64.

direction of the movement of the armature corresponds to the direction of the current in the winding. Thus, if the current flows through the winding in the direction of the arrows in Fig. 65 the armature will be attracted to the left.

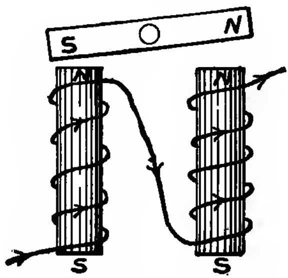


Fig. 65.

If now the current be passed through the windlng in the opposite direction, the armature will be attracted to the right. The same results will be obtained with the magnet in Fig. 64.

The principle lies in the fact that like poles repel one another, while unlike poles are attracted, therefore the armature is simultaneously attracted by one pole and repelled by the other.

The methods of polarizing the entire magnet, including the armature, which is soft iron in this case, are illustrated in Fig. 66 and Fig. 67, the former being used principally on telegraph instruments, and the latter on telephone signal bells. Both respond to alternating currents of low frequency, the synchronous action depending upon the inertia of the armature.

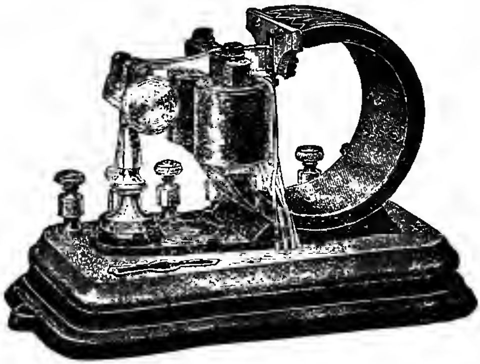


Fig. 66.

Still another form has the winding upon the armature which oscillates between permanent magnets.

Polarized magnets are very sensitive and may be worked with great rapidity.

Where direct currents are used, the armature may be just balanced by means of a spring, so that the least

change in the strength of the field will disturb the balance and move the armature.

The reason why polarized magnets are so much more sensitive than the non-polarized magnets is because there is a greater change in the flux density in the former than in the latter.

Consider a polarized electromagnet in a telephone receiver, assuming the flux density to be 5,000 lines per

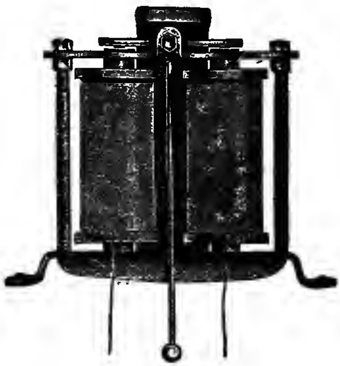


Fig. 67.

square inch due to the permanent magnet alone, and that the current in the winding increased it to 5,005. The pull on the diaphragm before the current flowed would be proportional to $B^2 = 5{,}000^2 = 25{,}000{,}000$, and after the current flowed, $B^2 = 5{,}005^2 = 25{,}050{,}025$, an increase in pull proportional to $25{,}050{,}025 - 25{,}000{,}000 = 50{,}025$, for a change in flux density due to the current of $5{,}005 - 5{,}000 = 5$ lines per square inch; whereas, if the magnet had not been polarized the increase in pull would only

have been $5^2 - 0 = 25$ for the same change in flux density, i.e., 5 lines per square inch. Therefore, the polarized magnet in this case would be $\frac{50025}{25} = 2{,}001$ times stronger than the electromagnet alone.

Since the permeability of the permanent magnet is very much less than that of a soft iron core, there is not so great a change in the flux in the steel as in the iron, but nevertheless the polarized magnet is many times more sensitive than one that is not polarized.

Problems.

105. How many pounds will a two-pole magnet lift whose pole areas are 1.5 square inches each, and the total useful flux is 140,000 lines? $P = 362.3$.

106. What would be the effect of increasing the area of each pole, assuming the total useful flux to be the same continually?

107. If the magnet in Problem 105 was polarized, i.e., if it had a continuous flux through its magnetic circuit, and the total useful flux due to the polarization was 100,000 lines, what would be the per cent increase in traction if the total useful flux was increased to 160,000 lines by means of the magnetizing coil? 156%.

CHAPTER V.

ELECTROMAGNETIC PHENOMENA.

63. Induction.

When a current of electricity flows through a wire or a coil of wire, lines of magnetic induction are established about the wire, or, in the case of the coil of wire, they pass through the center and about the coil.

Now, if a wire be passed through a magnetic field at an angle to the lines of force, a current of electricity will be generated in the wire. This is the principle of all dynamo electric machines. A current will also be generated in the wire, if it be placed in the magnetic field, and the field then disturbed or entirely destroyed, or suddenly increased from zero to maximum.

This action of a magnetic field upon a wire is called *Induction*, and is utilized in induction coils and transformers.

The induction between separate coils is called *Mutual Induction.*

64. Self-Induction.

The principle of *Self-Induction* or *Inductance* is the same as induction between a wire conducting a current and a wire placed near it, with the exception that the wire conducting the current is acted upon by the field produced by that current. Thus, whenever the current in the wire changes in intensity, the magnetic field also changes, and thus in turn generates another current in the wire.

If the current in the wire increases, the induced E.M.F. will be in the opposite direction, thus tending to retard or hold back the original current.

This effect only lasts, however, while the current is changing in intensity, so that as soon as the current is constant the self-induction ceases, and the current reaches its maximum.

Thus, when the current is switched on to the wire, it is retarded by an opposing E.M.F. produced in the wire, and therefore does not reach a maximum instantly.

Again, when the current is suddenly stopped, the induced E.M.F. acts in the opposite direction, i.e., in the same direction as the current. The effect is greatly increased when there is iron in the field, and an enormous amount of current flows through the wire producing a large spark at the point of rupture.

This principle is taken advantage of in electric gas-lighting apparatus where the self-induction is purposely made high in order to produce a strong, hot spark.

The E.M.F. induced in a coil of wire is one volt for each turn of wire when the lines of force increase or decrease at the rate of 100,000,000 per second.

The *henry* is the unit of induction or self-induction, and is represented by the symbol L.

An inductance of one henry gives rise to an E.M.F. of one volt when the current varies at the rate of one ampere per second.

The inductance is equal to the product of the number of turns of wire times the strength of the field, divided by the current, and also by 10^8 to bring it all under the practical system; thus,

$$L = \frac{\phi N}{I \times 10^8}. \tag{231}$$

L is a constant in a coil without iron, but is not a constant when iron is in the magnetic circuit, on account of the variable permeability of the iron under different degrees of magnetization.

In the design of quick-acting magnets, it is necessary to consider the effects due to inductance. When the circuit is closed the current does not immediately reach a maximum, but requires a certain length of time, and Ohm's law, $I = \frac{E}{\rho}$ (1), does not hold in this case. However, it may be expressed at the end of any short time, t, by Helmholtz's law,

$$I = \frac{E}{\rho}\left(1 - e^{-\frac{\rho}{L}t}\right), \tag{232}$$

where $e = 2.7182818$, which is the base of the Napierian logarithms.

The ratio $\frac{L}{\rho}$ is called the time constant of the magnet.

By substituting $\frac{L}{\rho}$ for t in (232),

$$I = \frac{E}{\rho}\left(\frac{e-1}{e}\right) \tag{233}$$

$$= .634\frac{E}{\rho}. \tag{234}$$

From the above it is seen that the time constant of a magnet is the time it takes for the current to reach .634 of its Ohm's-law value.

The time constant may be decreased by decreasing the inductance, or by increasing the electrical resistance,

since the E.M.F. will remain unchanged; therefore it is a small percentage of the total obstruction to the passage of the current, and the current reaches a given percentage of its final value sooner.

65. Alternating Currents.

With alternating currents the inductance retards the current, thus increasing the *Effective Resistance.* The effective resistance is called the *Impedence*, and is equal to

$$Z = \sqrt{\rho^2 + (2\,\pi L f)^2}, \qquad (235)$$

where L is the inductance in henries and f is the number of complete cycles per second, one cycle being two alterations or reversals of current.

The exact value of the current strength can not be easily calculated, due to the variable inductance, but if a curve be plotted showing the magnetic flux for each current strength, an accurate value of the current strength may be obtained.

On account of the inductance, an alternating-current electromagnet should contain less resistance than one for use with direct currents. An alternating-current-electromagnet should also be worked with lower densities of induction on account of the hysteresis losses.

66. Eddy Currents.

Another loss is due to the currents set up in the iron by induction. These currents are called *Eddy Currents*, and are largely overcome by subdividing the core in the direction of the lines of force.

For bar electromagnets, the core may consist of fine

wires, but for magnets of the horseshoe type, the cores are usually iron stampings, which should be insulated from one another by a coat of shellac or other insulating material.

In the design of these cores, great care must be exercised in the selection of the proper size of wire or laminæ, for if the wire or laminæ be too large in cross-section the loss due to eddy currents will be too great; on the other hand, if the wires be too small in cross-section, or the insulation be too thick, the magnetic reluctance will be so great as to more than offset the evil effects of the eddy currents.

APPENDIX

STANDARD COPPER WIRE TABLE.

(Copied from the "Supplement to Transactions of American Institute of Electrical Engineers," October, 1893.)

Giving Weights, Lengths, and Resistances of Wires @ 20° C. or 68° F., of Matthiessen's Standard Conductivity, for A. W. G. (Brown & Sharpe).

A.W.G.	DIAMETER	AREA.	WEIGHT.		LENGTH.		RESISTANCE.	
B. & S.	Inches.	Circ'l'r Mils.	Lbs. Per Foot	Lbs. Per Ohm.	Feet Per Lb.	Feet Per Ohm.	Ohms Per Lb.	Ohms Per Foot.
0000	0.460	211,600.	0.6405	13,090.	1.561	20,440.	0.00007639	0.00004893
000	0.4096	167,800.	0.5080	8,232.	1.969	16,210.	0.0001215	0.00006170
00	0.3648	133,100.	0.4028	5,177.	2.482	12,850.	0.0001931	0.00007780
0	0.3249	105,500.	0.3195	3,256.	3.130	10,190.	0.0003071	0.00009811
1	0.2893	83,690.	0.2533	2,048.	3.947	8,083.	0.0004883	0.0001237
2	0.2576	66,370.	0.2009	1,288.	4.977	6,410.	0.0007765	0.0001560
3	0.2294	52,630.	0.1593	810.0	6.276	5,084.	0.001235	0.0001967
4	0.2043	41,740.	0.1264	509.4	7.914	4,031.	0.001963	0.0002480
5	0.1819	33,100.	0.1002	320.4	9.980	3,197.	0.003122	0.0003128
6	0.1620	26,250.	0.07946	201.5	12.58	2,535.	0.004963	0.0003944
7	0.1443	20,820.	0.06302	126.7	15.87	2,011.	0.007892	0.0004973
8	0.1285	16,510.	0.04998	79.69	20.01	1,595.	0.01255	0.0006271
9	0.1144	13,090.	0.03963	50.12	25.23	1,265.	0.01995	0.0007908
10	0.1019	10,380.	0.03143	31.52	31.82	1,003.	0.03173	0.0009972
11	0.09074	8,234.	0.02493	19.82	40.12	795.3	0.05045	0.001257
12	0.08081	6,530.	0.01977	12.47	50.59	630.7	0.08022	0.001586
13	0.07196	5,178.	0.01568	7.840	63.79	500.1	0.1276	0.001999
14	0.06408	4,107.	0.01243	4.931	80.44	396.6	0.2028	0.002521
15	0.05707	3,257.	0.009858	3.101	101.4	314.5	0.3225	0.003179
16	0.05082	2,583.	0.007818	1.950	127.9	249.4	0.5128	0.004009
17	0.04526	2,048.	0.006200	1.226	161.3	197.8	0.8153	0.005055
18	0.04030	1,624.	0.004917	0.7713	203.4	156.9	1.296	0.006374
19	0.03589	1,288.	0.003899	0.4851	256.5	124.4	2.061	0.008038
20	0.03196	1,022.	0.003092	0.3051	323.4	98.66	3.278	0.01014
21	0.02846	810.1	0.002452	0.1919	407.8	78.24	5.212	0.01278
22	0.02535	642.4	0.001945	0.1207	514.2	62.05	8.287	0.01612
23	0.02257	509.5	0.001542	0.07589	648.4	49.21	13.18	0.02032
24	0.02010	404.0	0.001223	0.04773	817.6	39.02	20.95	0.02563
25	0.01790	320.4	0.0009699	0.03002	1,031.	30.95	33.32	0.03231
26	0.01594	254.1	0.0007692	0.01888	1,300.	24.54	52.97	0.04075
27	0.0142	201.5	0.0006100	0.01187	1,639.	19.46	84.23	0.05138
28	0.01264	159.8	0.0004837	0.007466	2,067.	15.43	133.9	0.06479
29	0.01126	126.7	0.0003836	0.004696	2,607.	12.24	213.0	0.08170
30	0.01003	100.5	0.0003042	0.002953	3,287.	9.707	338.6	0.1030
31	0.008928	79.70	0.0002413	0.001857	4,145.	7.698	538.4	0.1299
32	0.007950	63.21	0.0001913	0.001168	5,227.	6.105	856.2	0.1638
33	0.007080	50.13	0.0001517	0.0007346	6,591.	4.841	1,361.	0.2066
34	0.006305	39.75	0.0001203	0.0004620	8,311.	3.839	2,165.	0.2605
35	0.005615	31.52	0.00009543	0.0002905	10,480.	3.045	3,441.	0.3284
36	0.0050	25.0	0.00007568	0.0001827	13,210.	2.414	5,473.	0.4142
37	0.004453	19.83	0.00006001	0.0001149	16,660.	1.915	8,702.	0.5222
38	0.003965	15.72	0.00004759	0.00007210	21,010.	1.519	13,870.	0.6585
39	0.003531	12.47	0.00003774	0.00004545	26,500.	1.204	22,000.	0.8304
40	0.003145	9.888	0.00002993	0.00002858	33,410.	0.9550	34,980.	1.047

[For Explanatory Remarks on this Table, see next page.]

EXPLANATION OF TABLE.

The data from which this table has been computed are as follows: — Matthiessen's standard resistivity, Matthiessen's temperature coefficients, specific gravity of copper = 8.89. Resistance in terms of the international ohm.

Matthiessen's standard 1 metre-gramme of hard drawn copper = 0.1469 B.A.U. @ 0° C. Ratio of resistivity hard to soft copper 1.0226.

Matthiessen's standard 1 metre-gramme of soft drawn copper = 0.14365 B.A.U. @ 0° C. One B.A.U. = 0.9866 international ohms.

Matthiessen's standard 1 metre-gramme soft drawn copper = 0.141729 international ohm @ 0° C.

Temperature coefficients of resistance for 20° C., 50° C., and 80° C., 1.07968, 1.20625, and 1.33681 respectively. 1 foot = 0.3048028 metre, 1 pound = 453.59256 grammes.

Although the entries in the table are carried to the fourth significant digit, the computations have been carried to at least five figures. The last digit is therefore correct to within half a unit, representing an arithmetical degree of accuracy of at least one part in two thousand. The diameters of the B. & S. or A. W. G. wires are obtained from the geometrical series in which No. 0000 = 0.4600 inch and No. 36 = 0.005 inch, the nearest fourth significant digit being retained in the areas and diameters so deduced.

It is to be observed that while Mathiessen's standard of resistivity may be permanently recognized, the temperature coefficient of its variation which he introduced, and which is here used, may in future undergo slight revision.

F. B. CROCKER,
G. A. HAMILTON,
W. E. GEYER,
A. E. KENNELLY, *Chairman*,
} *Committee on "Units and Standards."*

Pounds per foot varies directly as the area.
Pounds per ohm varies directly as the area squared.
Feet per pound varies inversely as the area.
Feet per ohm varies directly as the area.
Ohms per pound varies inversely as the area squared.
Ohms per foot varies inversely as the area.

BARE COPPER WIRE.

B. & S. No.	Δ.	Δ².	K.	Ω.
10	.1019	.01038	.000261	.03173
11	.09074	.008234	.000329	.05045
12	.08081	.006530	.000415	.08022
13	.07196	.005178	.000523	.1276
14	.06408	.004107	.00066	.2028
15	.05707	.003257	.000833	.3225
16	.05082	.002583	.001049	.5128
17	.04526	.002048	.001323	.8153
18	.04030	.001624	.001668	1.296
19	.03589	.001288	.002103	2.061
20	.03196	.001022	.00265	3.278
21	.02846	.000810	.00335	5.212
22	.02535	.0006424	.00422	8.287
23	.02257	.0005095	.00532	13.18
24	.0201	.000404	.00671	20.95
25	.0179	.0003204	.00846	33.32
26	.01594	.0002541	.01065	52.97
27	.0142	.00020015	.01345	84.23
28	.01264	.0001598	.01696	133.9
29	.01126	.0001267	.0214	213.0
30	.01003	.0001005	.02695	338.6
31	.008928	.0000797	.034	538.4
32	.00795	.00006321	.0429	856.2
33	.00708	.00005013	.0541	1361
34	.006305	.00003975	.0682	2165
35	.005615	.00003152	.086	3441
36	.005	.000025	.10845	5473
37	.004453	.00001983	.1365	8702
38	.003965	.00001572	.1725	13,870
39	.003531	.00001247	.217	22,000
40	.003145	.000009888	.2741	34,980

BARE COPPER WIRE.

COMMERCIAL HALF SIZES.

B. & S. No.	Δ.	Δ².	K.	Ω.
30½	.0095	.00009025	.03	420
31½	.0085	.0000723	.0375	655
32½	.0076	.0000578	.0469	1025
33½	.0067	.0000449	.0604	1695
34½	.0060	.0000360	.0753	2640
35½	.0054	.0000292	.093	4020
36½	.0048	.00002305	.1176	6440
37½	.0042	.00001765	.1536	11000
38½	.0038	.00001445	.1877	16400
39½	.0033	.00001090	.249	28800
40½	.0030	.000009	.301	42250

WEIGHT OF COPPER IN 100 POUNDS OF COTTON COVERED WIRE.

B. & S.	5 Mil.	10 Mil.	B. & S.	4 Mil.	8 Mil.	B. & S.	4 Mil.	8 Mil.
10	98.5	97	20	96.2	92.2	30	87	74.5
11	98.25	96.5	21	95.5	90.8	31	85.5	71.3
12	98	96	22	95	89.8	32	83.7	68.3
13	97.75	95.5	23	94.3	88.6	33	81.8	64.7
14	97.5	95	24	93.7	87	34	79.4	61
15	97.25	94.5	25	92.8	85.5	35	76.8	57
16	97	93.7	26	91.8	83.8	36	74.3	52.8
17	96.5	93.1	27	90.8	81.7	37	71.2	48.7
18	96	92	28	89.7	79.5	. .	. . .	. .
19	95.5	91.1	29	88.5	77	. .	. .	. . .

WEIGHT OF COPPER IN 100 POUNDS OF COTTON COVERED WIRE.

COMMERCIAL HALF SIZES.

B. & S.	4 Mil.	8 Mil.
30½	86.5	73
31½	85	70
32½	83	66.8
33½	80.5	63
34½	78.5	59.3
35½	76.1	55.6
36½	73.2	51.3

WEIGHT OF COPPER IN 100 POUNDS OF SILK INSULATED WIRE.

COMMERCIAL HALF SIZES.

B. & S.	2 Mil.	4 Mil.
30½	95	89.5
31½	94.5	88.3
32½	93.7	86.8
33½	92.7	84.7
34½	91.8	82.9
35½	91	81
36½	89.7	78.5
	1.5 Mil.	3 Mil.
37½	91.1	81.7
38½	90.3	79.7
39½	88.5	76.6
40½	87.5	74.1

WEIGHT OF COPPER IN 100 POUNDS OF SILK INSULATED WIRE.

B. & S.	1.5 Mil.	2 Mil.	3 Mil.	4 Mil.
20	99	98.5	97.8	97
21	98.8	98.3	97.5	96.5
22	98.6	98	97.2	96.2
23	98.4	97.8	96.8	95.8
24	98.2	97.6	96.3	95.2
25	98	97.2	95.8	94.6
26	97.8	97	95.3	93.8
27	97.5	96.7	94.8	93
28	97.2	96.2	94.2	92.2
29	96.8	95.7	93.4	91.2
30	96.3	95.2	92.6	90.6
31	95.8	94.6	91.7	88.6
32	95.4	93.8	90.5	87.3
33	94.8	93	89.25	85.7
34	94.2	92.2	87.8	83.7
35	93.4	91.2	86.7	82.2
36	92.6	90	84.7	79.3
37	91.6	88.8	82.6	76.7
38	90.5	87.2	80.4	73.8
39	89.2	85.7	77.2	70.8
40	88.1	83.8	75.5	67.7

INSULATED WIRE TABLES

COMPUTED FROM THE FOLLOWING DATA *

Size of Wire (B. & S.).	Diameter of Wire (Bare).	Diameter insulated with Silk.	Diameter insulated with Cotton.	Weight of Product.	Weight of Silk.	Weight of Cotton.
29	.01126	.01326	....	104.5	4.5	....
29	.01126	....	.01526	112.95	....	12.95

* R. Varley.

10 MIL. DOUBLE COTTON, INSULATED WIRE.

B. & S. No.	g.	g^2.	R.	θ.	w.
10	.11190	.0125	.0209	.0308	.679
11	.10074	.01015	.0324	.04865	.665
12	.09081	.00825	.0503	.077	.653
13	.08196	.00672	.0779	.1219	.640
14	.07408	.00550	.12	.1928	.623
15	.06707	.00450	.185	.3045	.608
16	.06082	.00370	.2835	.4800	.591
17	.05526	.003055	.434	.7600	.571
18	.05030	.00253	.66	1.192	.553
19	.04589	.002105	.999	1.880	.532

8 MIL. DOUBLE COTTON.

B. & S. No.	g.	g^2.	R.	θ.	w.
20	.03996	.001597	1.66	3.02	.55
21	.03646	.00133	2.52	4.73	.533
22	.03335	.001112	3.79	7.445	.51
23	.03057	.000935	5.69	11.68	.487
24	.02810	.000787	8.525	19.05	.448
25	.02590	.000671	12.60	28.45	.443
26	.02394	.000574	18.55	44.35	.418
27	.02220	.000493	27.25	68.8	.396
28	.02064	.000426	39.80	106.4	.374
29	.01926	.000371	57.65	164	.352
30	.01803	.000325	82.92	252	.329
31	.016928	.0002865	118.6	384	.309
32	.015950	.0002545	168.5	585	.288
33	.015080	.0002275	238	882	.27
34	.014305	.000205	333	1,320	.252
35	.013615	.0001855	464	1,960	.237
36	.013000	.000169	642	2,890	.222
37	.012453	.0001551	880	4,237	.208

5-MIL. SINGLE COTTON.

B. & S. No.	g.	g^2.	R.	θ.	w.
10	.1069	.01142	.02285	.0312	.734
11	.09574	.00917	.0359	.0495	.726
12	.08581	.00737	.0563	.0766	.716
13	.07696	.00593	.0882	.1249	.706
14	.06908	.00477	.1385	.198	.700
15	.06207	.00385	.2160	.304	.711
16	.05582	.00312	.3365	.497	.678
17	.05026	.002525	.5250	.787	.667
18	.04530	.002055	.8125	1.245	.653
19	.04089	.00167	1.260	1.968	.64

4-MIL. SINGLE COTTON.

B. & S. No.	g.	g^2.	R.	θ.	w.
20	.03596	.001293	2.05	3.15	.65
21	.03246	.001053	3.18	4.97	.64
22	.02935	.000862	4.895	7.87	.622
23	.02657	.000705	7.55	12.45	.606
24	.0241	.000580	11.56	19.65	.59
25	.0219	.000479	17.66	30.9	.572
26	.01994	.000397	26.86	48.5	.554
27	.0182	.000332	40.5	76.5	.530
28	.01664	.000277	61.2	120	.510
29	.01526	.000233	91.8	190.5	.482
30	.01403	.000197	136.8	294.5	.464
31	.012928	.000167	203.5	461	.441
32	.01195	.000143	299.8	717	.418
33	.01108	.000123	439.5	1,115	.394
34	.010305	.000106	643	1,715	.375
35	.009615	.0000925	930	2,640	.352
36	.009	.0000810	1,340	4,070	.329
37	.008453	.0000714	1,912	6,180	.309

8-MIL. DOUBLE COTTON.

COMMERCIAL HALF SIZES.

No.	Δ.	*g*.	g^2.	*R*.	θ.	*w*.
30½	.0095	.0175	.000306	98	306	.32
31½	.0085	.0165	.000272	138	459	.30
32½	.0076	.0156	.0002435	192.5	685	.281
33½	.0067	.0147	.000216	280	1,068	.262
34½	.0060	.0140	.000196	384	1,565	.246
35½	.0054	.0134	.000180	517	2,235	.231
36½	.0048	.0128	.000164	717	3,300	.217

4-MIL. SINGLE COTTON.

COMMERCIAL HALF SIZES.

No.	Δ.	*g*.	g^2.	*R*.	θ.	*w*.
30½	.0095	.0135	.000182	165	363.5	.455
31½	.0085	.0125	.000156	240	556	.431
32½	.0076	.0116	.000135	348	850	.410
33½	.0067	.0107	.000115	526	1,363	.386
34½	.0060	.0100	.000100	753	2,070	.364
35½	.0054	.0094	.0000885	1,050	3,060	.343
36½	.0048	.0088	.0000775	1,520	4,710	.322

4-MIL. DOUBLE SILK.

No.	g.	g^2.	R.	θ.	w.
20	.03596	.001293	2.05	3.175	.645
21	.03246	.001053	3.18	5.025	.633
22	.02935	.000862	4.895	7.96	.615
23	.02657	.000705	7.55	12.65	.597
24	.0241	.000580	11.56	19.95	.58
25	.0219	.000479	17.66	31.5	.56
26	.01994	.000397	26.86	49.7	.54
27	.0182	.000332	40.5	78.3	.518
28	.01664	.000277	61.2	123.5	.495
29	.01526	.000233	91.8	194	.473
30	.01403	.000197	136.8	306.5	.446
31	.012928	.000167	203.5	477	.426
32	.01195	.000143	299.8	747	.402
33	.01108	.000123	439 5	1,165	.378
34	.010305	.000106	643	1,810	.356
35	.009615	.0000925	930	2,820	.33
36	.009	.000081	1,340	4,340	.309

3-MIL. DOUBLE SILK.

No.	g.	g^2.	R.	θ.	w.
37	.007453	.0000555	2,460	7,180	.343
38	.006965	.0000485	3,560	11,150	.319
39	.006531	.0000426	5,100	17,000	.3
40	.006145	.0000378	7,260	26,400	.275

2-MIL. SINGLE SILK.

No.	g.	g^2.	R.	θ.	w.
20	.03396	.001152	2.3	3.23	.713
21	.03046	.000928	3.61	5.125	.705
22	.02735	.000748	5.64	8.12	.695
23	.02457	.000604	8.8	12.90	.682
24	.02210	.000487	13.8	20.45	.675
25	.01990	.000396	21.4	32.4	.661
26	.01794	.000322	33.1	51.3	.645
27	.01620	.0002625	51.2	81.4	.63
28	.01464	.000214	79.3	129	.615
29	.01326	.000176	121.5	204	.596
30	.01203	.000145	186	322	.578
31	.010928	.0001195	284	510	.557
32	.009950	.000099	434	803	.54
33	.009080	.0000825	656	1,265	.52
34	.008305	.000069	990	1,995	.497
35	.007615	.000058	1,480	3,140	.472
36	.007	.000049	2,210	4,926	.45

1.5-MIL. SINGLE SILK.

No.	g.	g^2.	R.	θ.	w.
37	.005953	.0000354	3,860	7,970	.484
38	.005465	.0000299	5,770	12,550	.459
39	.005031	.0000253	8,590	19,600	.438
40	.004645	.0000216	12,690	30,800	.412

4-MIL. DOUBLE SILK.

COMMERCIAL HALF SIZES.

No.	Δ.	g.	g^2.	R.	θ.	w.
30½	.0095	.0135	.000182	165	376	.44
31½	.0085	.0125	.000156	240	578	.415
32½	.0076	.0116	.000135	348	890	.392
33½	.0067	.0107	.000115	526	1,435	.367
34½	.0060	.0100	.000100	753	2,190	.344
35½	.0054	.0094	.0000885	1,050	3,255	.323
36½	.0048	.0088	.0000775	1,520	5,050	.301

3-MIL. DOUBLE SILK.

COMMERCIAL HALF SIZES.

No.	Δ.	g.	g^2.	R.	θ.	w.
37½	.0042	.0072	.0000518	2,960	9,000	.329
38½	.0038	.0068	.0000462	4,060	13,050	.311
39½	.0033	.0063	.0000397	6,275	22,050	.284
40½	.0030	.0060	.0000360	8,360	31,300	.267

2-MIL. SINGLE SILK.

COMMERCIAL HALF SIZES.

No.	Δ.	g.	g^2.	R.	θ.	w.
30½	.0095	.0115	.000132	227	399	.57
31½	.0085	.0105	.000110	341	619	.55
32½	.0076	.0096	.0000922	509	960	.53
33½	.0067	.0087	.0000757	798	1,570	.51
34½	.0060	.0080	.0000640	1,176	2,420	.487
35½	.0054	.0074	.0000548	1,695	3,660	.464
36½	.0048	.0068	.0000462	2,550	5,770	.442

1.5-MIL. SINGLE SILK.

COMMERCIAL HALF SIZES.

No.	Δ.	g.	g^2.	R.	θ.	w.
37½	.0042	.0057	.0000325	4,730	10,020	.472
38½	.0038	.0053	.0000281	6,680	14,800	.452
39½	.0033	.0048	.0000231	10,800	25,450	.425
40½	.0030	.0045	.0000203	14,850	37,000	.402

TABLE OF RESISTANCES OF GERMAN SILVER WIRE.

SPECIFIC GRAVITY 8.5 (APPROX.).

Size B. & S.	18% Alloy. Specific Resistance, 29.45.		30% Alloy. Specific Resistance, 44.18.	
	Ohms per 1,000 Feet.	Ohms per Pound.	Ohms per 1,000 Feet.	Ohms per Pound.
8	11.77	.2470	17.66	.3705
9	14.83	.3925	22.22	.5887
10	18.72	.6244	28.08	.9367
11	23.60	.9928	35.40	1.489
12	29.75	1.579	44.63	2.368
13	37.51	2.510	56.27	3.765
14	47.30	3.991	70.96	5.986
15	59.65	6.346	89.48	9.519
16	75.22	10.09	112.8	15.14
17	94.84	16.04	142.3	23.52
18	119.6	25.51	179.4	38.27
19	155.1	42.91	232.7	64.36
20	190.2	64.50	285.3	96.75
21	239.8	102.6	359.7	153.8
22	302.4	163.1	453 6	244.6
23	381.3	259.3	572.0	389.0
24	480.8	412.4	721.3	618.6
25	606.3	655.6	909.5	983.4
26	764.6	1,043	1,147	1,564
27	964.1	1,658	1,446	2,487
28	1,216	2,636	1,824	3,954
29	1,533	4,192	2,300	6,287
30	1,933	6,651	2,900	10,000
31	2,437	10,590	3,656	15,890
32	3,074	16,850	4,611	25,280
33	3,876	26,790	5,813	40,180
34	4,888	42,620	7,333	63,930
35	6,164	67,760	9,246	101,600
36	7,771	107,700	11,660	161,500
37	9,797	171,200	14,700	256,700
38	12,360	269,800	18,540	404,800
39	15,570	428,700	23,360	644,600
40	19,650	682,500	29,480	1,024,000

PERMEABILITY TABLE.

Density of Magnetization.		Permeability, μ.			
B Lines per Square Inch.	$\mathfrak{B}$ Lines per Square Centimetre.	Annealed Wrought Iron.	Commercial Wrought Iron.	Gray Cast Iron.	Ordinary Cast Iron
20,000	3,100	2,600	1,800	850	650
25,000	3,875	2,900	2,000	800	700
30,000	4,650	3,000	2,100	600	770
35,000	5,425	2,950	2,150	400	800
40,000	6,200	2,900	2,130	250	770
45,000	6,975	2,800	2,100	140	730
50,000	7,750	2,650	2,050	110	700
55,000	8,525	2,500	1,980	90	600
60,000	9,300	2,300	1,850	70	500
65,000	10,100	2,100	1,700	50	450
70,000	10,850	1,800	1,550	35	350
75,000	11,650	1,500	1,400	25	250
80,000	12,400	1,200	1,250	20	200
85,000	13,200	1,000	1,100	15	150
90,000	14,000	800	900	12	100
95,000	14,750	530	680	10	70
100,000	15,500	360	500	9	50
105,000	16,300	260	360	..	...
110,000	17,400	180	260	...	...
115,000	17,800	120	190	...	...
120,000	18,600	80	150	..	..
125,000	19,400	50	120	..	...
130,000	20,150	30	100	...	..
135,000	20,900	20	85	...	..
140,000	21,700	15	75	..	...

TRACTION TABLE.

B Lines per Square Inch.	Traction in Pounds per Square Inch.	B Lines per Square Inch.	Traction in Pounds per Square Inch.
10,000	1.386	75,000	77.99
15,000	3.119	80,000	88.72
20,000	5.545	85,000	100.1
25,000	8.664	90,000	112.3
30,000	12.48	95,000	125.1
35,000	16.98	100,000	138.6
40,000	22.18	105,000	152.8
45,000	28.07	110,000	167.8
50,000	34.66	115,000	183.3
55,000	41.93	120,000	199.6
60,000	49.91	125,000	210.6
65,000	58.57	130,000	234.3
70,000	67.93	. . . .	. . . .

INSULATING MATERIALS.*

Material.	Grade.	Thickness in Mils.	Puncturr Test in Volts.	Guaranteed Resistance to Puncture.
Linen	A	6–7	5,000– 9,000	. . . .
Linen	B	8–9	13,000–15,000	10,000
Linen	C	11–12	18,000–23,000	15,000
Insulated canvas .	. . .	10–11	5,000– 9,000	. . . .
Paper	A	5–6	8,000–10,000	. . . .
Paper	B	8–9	14,000–16,000	10,000
Paper	C	11–12	20,000–25,000	15,000
Bond paper . . .	A	4–5	5,000– 9,000	. . . .
Fiber paper . . .	A	5–6	8,000–10,000	. . . .
Red rope paper .	A	8–9	9,000–11,000	. . . .

* Pittsburgh Insulating Company.

8ths.	16ths.	32ds.	64ths.	Decimal Equivalent
			1..	.015625
		1..		.03125
			3..	.046875
	1..			.0625
			5..	.078125
		3..		.09375
			7..	.109375
1				.125
			9..	.140625
		5		.15625
			11..	.171875
	3..			.1875
			13..	.203125
		7..		.21875
			15..	.234375
				.25
			17..	.265625
		9..		.28125
			19..	.296875
	5			.3125
			21..	.328125
		11..		.34375
			23..	.359375
3.				.375
			25..	.390625
		13..		.40625
			27..	.421875
				.4375
			29..	.453125
		15..		.46875
			31..	.484375
				.500000
			33..	.515625
		17		.53125
			35..	.546875
	9..			.5625
			37	.578125
		19..		.59375
			39	.609375
5				.625
			41..	.640625
		21..		.65625
			43..	.671875
	11..			.6875
			45..	.703125
		23..		.71875
			47..	.734375
				.75
			49..	.765625
		25..		.78125
			51..	.796875
	13..			.8125
			53..	.828125
		27..		.84375
			55..	.859375
7..				.875
			57..	.890625
		29..		.90625
			59..	.921875
	15..			.9375
			61..	.953125
		31..		.96875
			63..	.984375

LOGARITHMS OF NUMBERS.

Natural Numbers.	0	1	2	3	4	5	6	7	8	9	Proportional Parts. 1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS OF NUMBERS.

Natural Numbers.	0	1	2	3	4	5	6	7	8	9	Proportional Parts. 1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS.

Logarithms.	0	1	2	3	4	5	6	7	8	9	Proportional Parts. 1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3156	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS.

Loga-rithms.	0	1	2	3	4	5	6	7	8	9	Proportional Parts. 1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

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